

Evolution of Signaling in Adaptive Networks

Diogo Luís Neves dos Santos Pires

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Supervisors: Prof. Francisco João Duarte Cordeiro Correia dos Santos
Prof. José Luís Rodrigues Júlio Martins

Examination Committee

Chairperson: Prof. Ilídio Pereira Lopes
Supervisor: Prof. Francisco João Duarte Cordeiro Correia dos Santos
Member of the Committee: Dr. Fernando Pedro Pascoal dos Santos

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Resumo

A auto-organização de sinalização é um tópico de interesse em teoria evolutiva, devido à sua presença ubíqua na natureza. Aqui mostramos que a coevolução de estratégias de sinalização e da estrutura de interação de uma população poderá constituir um mecanismo central na evolução e sustentabilidade de sinalização informativa. Com esse fim, é proposto um novo modelo estocástico de dinâmicas populacionais — caracterizado por uma cadeia de Markov reduzida — que nos permite determinar a distribuição estacionária de estratégias de sinalização em redes adaptativas. Mostra-se que a estrutura desenvolvida através seleção de parceiros baseada em sinais reforça a evolução de sinalização, tanto na ausência como na presença de conflito parcial, aqui definido por indivíduos beneficiarem da realização de ações diferentes. Dependendo do nível de conflito, tanto é observada a predominância de sinalização informativa na dinâmica populacional, como a sua ocorrência numa inter-relação complexa com estratégias desonestas. Estes resultados mantêm-se num intervalo vasto de parâmetros e sob diversas motivações individuais subentendidas na evolução temporal da rede.

Palavras-chave: sinalização, conflito, populações finitas, redes de interação adaptativas, seleção de parceiros

Abstract

The self-organization of signaling has been a topic of interest in evolutionary theory, due to its ubiquitous presence in nature. Here we show that the entangled evolution of signaling strategies and interaction structure may constitute a key mechanism for the evolution and sustainability of informative signaling. To this end, we propose a novel stochastic population dynamics model — described as a reduced Markov process — which allows us to compute the stationary distribution of signaling strategies in adaptive networks. We show that signal-based partner choice may lead to the evolution of informative signaling, both in the absence and presence of partial conflict, here associated with individuals sometimes benefiting from different actions when interacting. Depending on the level of conflict, signaling can either dominate the population dynamics or arise from a non-trivial interplay with deceivers. These results are shown to remain valid for a wide range of parameters and diverse individual motivations underpinning the time-evolution of the network.

Keywords: signaling, conflict, finite populations, adaptive interaction networks, partner choice

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List of Abbreviations

GT	Game Theory
EGT	Evolutionary Game Theory
ESS	Evolutionarily Stable Strategy
ERS	Evolutionarily Robust Strategy
SG	Signaling Game
SML	Small Mutation Limit
PCN	Partner Choice Norm
SS	Signaling System
D	Deceiver
S	Sender
R	Receiver
N	Nature
A	Adopter
NA	Non-Adopter
I1/2	Informative 1/2
UI	Uninformative

Nomenclature

S_i	State i
M_j	Signal j
A_k	Act k
β	Intensity of Selection
μ	Mutation Probability
π	Payoff
Π	Fitness
Z	Population Size
δ	Degree of Discrimination
a	Basic Payoff Parameter
b	Partial Conflict Parameter
b/a	Conflict Level
α	Linking Propensity
γ	Link's Death Rate

Chapter 1

Introduction

Information transfer is a ubiquitous phenomenon, occurring at all levels of complexity, from micro-organisms to humans [1–3]. The fundamental process that supports it is what is usually called signaling: the process through which signal usage carries information about the state of the world. However, the occurrence of signaling systems is nuanced and far from trivial. What is their nature? How do conventions concerning the use of the signals emerge? Here we will tackle the evolution of signaling, focusing particularly on the role of time-evolving interaction structures.

It can be tempting to look at signaling systems as a set of static conventions or agreements, in which it is implicitly defined which signals make sense and what is their exact meaning. However, a signaling system should not demand such a formal understanding, since that would require a preexisting language from its users. Focusing on the interactive nature of signaling, a formal approach was initiated by Lewis [4] and borrowed from what at the time was considered the economic field of game theory [5] to create the signaling game. Signaling games are characterized by the existence of two agents: one that has access to private information about the state of the world and signals contingent on it; and another with enough agency to act based on the signals received [4, 6]. On this basis, instead of signals having any innate meaning, they may earn one from usage.

This approach was then extended by the inclusion of evolutionary population dynamics [6–14], inspired by biology. These tools allowed advances in predicting and understanding emergent behavior in evolving populations of interacting agents. In what concerns signaling, the results obtained here seem to depend critically on the agents' interests. On one hand, if agents in a population benefit from the same actions, they are said to have common interests, and informative signaling is consistently predicted to emerge [15–19]. On the other hand, agents might be in the presence of partial conflict if they benefit from different actions under part of the states of the world.

Conflict of this form is as much present in real complex systems as signaling itself, some of the most famous examples being nestling feeding, mating quality, and job-market signaling (see these and more examples in [20] for nature, and [21] for economics). This is not trivially explained under evolutionary theory, where several honesty-sustaining features have been proposed to be present, such as costly signals [22–29], kinship [26, 30–32], partial information transfer [31–33], and reciprocity [34]. Most of the liter-

ature focuses on the first, despite the significant critiques regarding the lack of incidence of the required high costs on natural systems [35–38].

Contrary to this, time-evolving population structure [1, 39, 40] has been almost systematically neglected in past signaling models where populations are commonly considered static and well-mixed (an exception under deterministic dynamics being [41]). This comes off as surprising due to the fact that adaptive networks have been repeatedly pointed out as pervasive over all sorts of complex systems [42–46]. Because of this, signaling is a phenomenon whose intricate dynamics may not be completely disclosed, especially relating the impact of time-evolving population structure on its emergence.

We will be resorting to a set of analytical tools, based on a game-theoretic approach. We start by proposing a signaling game with a novel unifying payoff framework, which will allow the inclusion of partial conflict in the extensively explored Lewis signaling game. We will use stochastic methods on finite populations, an approach commonly overlooked to deterministic models and their stability concepts [7, 9, 10], but already shown to reveal other facets of emerging phenomena [47, 48]. Considering a population of agents who interact with each other under the basis of the signaling game, we will use as a stochastic update rule the pairwise comparison [11, 49], inspired by the Fermi function and statistical physics. The effect of mutations is taken into consideration, but because of the large size of the state space of the population, we will further use the small mutation limit [12]. By computing the transitions between monomorphic states and consequently, the stationary distribution of the population over them [12, 48, 50–53], we will achieve the values of signaling prevalence. The effect of adaptive interaction networks is considered by using the active linking framework [41, 52, 54–56]. We will develop a partner choice model, which will handily restrain the dynamical parameters of the network. This will allow us to focus on signal-based partner choice norms and briefly explore the potential of outcome-based ones.

Using this framework, we propose to answer the following questions:

- Does signaling emerge under partial conflict?
- What are the main mechanisms affecting the emergence of signaling?
- Can time-evolving population structure generated by partner choice enhance the emergence of signaling?
- Can partner choice emerge from natural selection?

To this end, in chapter 2, we start by motivating signaling and formalizing its interactive and strategic features. We elaborate on the problems created by conflicting interests and detail some of the mechanisms that can sustain signaling emergence under it. In chapter 3, we thoroughly outline the analytic framework to be used, as well as develop the novel contributions to the modeling of signaling (sections 3.1 and 3.4). In chapter 4, we provide a discussion of the obtained results for the evolution of signaling under conflict and the effects of considering adaptive interaction networks. In chapter 5, we design an extension of the initial model in order to accommodate the co-evolution of partner choice behavior and discuss the results obtained under this. Finally, in chapter 6, the main achievements of this work are outlined, as well as new research prospects arising from this work and possibly being carried out in the future.

Chapter 2

Background

In this chapter, we provide an overview of the theoretical foundations required to go through the present work. In section 2.1, we will start by elaborating on the presence of signaling across several complex systems, as a way of reaffirming and deepening the motivation given in the Introduction. Then in sections 2.2 and 2.3, we will outline a mathematical representation of signaling, developing at the same time the basis to understand why its emergence is not trivial. Finally, in section 2.4, we go over some of the mechanisms proposed to sustain signaling emergence under conflict.

2.1 Signaling

Signaling can be seen as the fundamental process behind information transfer. Signals are associated with certain states of affairs and by being used, transmit information about their incidence. In the Introduction, we have briefly mentioned some settings where signaling systems can be dismantled and analyzed. We now go back there and attempt to develop a stronger motivational ground for the background search that will be done in the following sections of this chapter, and ultimately for the work we will be developing in the chapters to be succeeded. We will start by looking into biological examples, on which we followed [1, Ch. 2] and [20, Ch. 2,3], then lean into economic ones following [21], and finally state the significance of signaling from a general standpoint.

Alarm Calls are one of the most studied signaling systems in nature. A large number of species use signals to warn about existing predators so others can be prepared. Some of these species, between them monkeys [57], lemurs [58], and prairie dogs [59], are reported to use different signals, each carrying particular information about the predator to be warned. Black-capped chickadees go beyond that and use complex signals, constructed through the juxtaposition of individual ones [60]. These complex signals are not limited, therefore having an infinite number of possible combinations, and may also be wrongly formulated, in which case they are just ignored.

Food foraging inside bee colonies has been another of the most extensively studied biological systems by signaling scholars, granting a Nobel prize to Karl von Frisch in 1973. Bees show very complex signals during that process, involving a waggle dance, to convey information to other bees from their hive about

the direction and distance of food found [61].

Nonetheless, signals in the biological world are not exclusive to animals. *Myxococcus xanthus* bacteria are one of the numerous species that we have observed to do it effectively through the use of chemicals [62]. When facing nutrient scarcity, they diffuse a particular chemical signal, which is collectively perceived by their swarm. The mass emission of this signal then triggers a surprising collective response, where the whole swarm switches from an individual mode of living to a temporary multicellular organism through differentiation.

All these signaling systems deserve the shown attention, but looking at them we see that they develop in extremely cooperative settings, either involving kin or closed communities. This means that all agents have common interests and ultimately that the successful transmission of all the information at hand benefits everyone individually. That assumption is not always appropriate. What if the signaling agents do not know each other? Or they do know each other, but when signaling they are in such competitive settings, that it is no longer appropriate to assume that they benefit from the same actions to be performed on the world? It seems counter-intuitive that these systems are sustainable, but as we will see next, there are several compelling examples.

One of the most pervasive cases in nature is nestling feeding [26, 63]. In most bird species, when progenitors find food, they can either eat the food themselves or feed it to their offspring. To signal their needs, the offspring may chirp. In this setting, it is always in the interest of the offspring to be fed, while it would be better for the progenitors to feed them only if they were hungry. Even though there is an evident conflict under a particular state of affairs, which could compromise the existence of honest signaling, this is nonetheless seen to occur over a vast number of species [64–66].

Another notoriously general setting is sexual selection. When facing the possibility of mating, specimens, usually the females, look for the quality of their potential mates. Sometimes this information is directly observed, but other times this is not possible. In these circumstances, their potential mates can resort to signals to convey information about their quality. However, while they always prefer to be seen as high-quality specimens in order to be chosen to mate, the one choosing is interested in mating only if that is true. Despite the existence of this partial conflict of interests under complete information transfer during sexual selection, honest signaling is still observed. This is explored in [20, Ch. 3], where three pervasive quality signals are examined: “carotenoids in birds and fish, song in songbirds, and long tails in birds”.

Having explored the existence of signaling systems in the natural world, we can extend the search outside its boundaries. There is a significant amount of research focusing on signaling systems where agents are composed of human institutions. We have three notable examples of these systems taken from [21], initially studied with the purpose of aiding in the process of optimal decision making.

Spence was the pioneer in this sort of procedure when he originally analyzed job-market signaling [27]. By dismantling the interaction present in a job interview, Spence looked at potential employees stating their capabilities or education as if they were sending signals. The question he then asked was when should we expect those signals to carry meaning? Or in other words, when should signaling systems emerge under this setting?

The model proposed in [28] assessed a similar mechanism under the context of general project proposals to shareholders by managers looking for investment. The value publicly attributed to the project is seen as a signal sent by the manager, which can be meaningful or not. This model is alternatively interpreted [21] as the search for project investment by entrepreneurs through the offer of an equity stake to potential investors.

Finally, in [29] we see another signaling model contemplating the decision-making behind monetary authorities. These may choose between properly signal the inflation level, which will lead to high enough wages, or reassuring low unemployment levels through inflation higher than signaled (surprise inflation).

There is a vast number of other cases where signaling processes are interesting, and sometimes very useful to be dismantled. But if they are already pervasive over biology and economic studies, they are even more central to human behavior. Communication is commonly seen as one of the reasons for human cooperation, the prosperity of the human species, and the development of our collective identity. Its origins are a topic that has been intriguing us for a long time, finding its deepest advances in the last centuries through philosophy, sociology, anthropology, history, evolutionary theory, etc.

It is in 1969 that David Lewis lays the foundation [4] for the angle we are aiming for. He completely rejects the need for established linguistic conventions and asks how do we come to attain what looks like them? This question can be extended to all the systems where we see signaling, including the ones we have previously elaborated under the scope of biology and economics. Lewis has answered it redefining apparent conventions as necessary outcomes of a common interest in exchanging information, rather than ordinary agreements of meaning. In his own words [4] “Convention turns out to be a general sense of common interest; which sense all the members of the society express to one another, and which induces them to regulate their conduct by certain rules. (...) And this may properly enough be called a convention or agreement betwixt us, though without the interposition of a promise; since the actions of each of us have a reference to those of the other, and are performed upon the supposition that something is to be performed on the other part.”

Here we include an extra layer of difficulty to this question, by relaxing the assumption that agents need to have absolute common interests during communication. As we have stated, honest signaling seems to co-exist ubiquitously with some level of partial conflict. Due to the complexity of the interaction between agents in signaling settings, we can almost always reveal the existence of conflicts created by parallel interests that could mine honesty and encourage deceiving behavior [57, 67, 68]. What are the origins of signaling under conflict? This is the primary question which we intend to answer under this work.

2.2 Game Theory

Decision-making processes have been intriguing us and stimulating the constant development of new approaches for decades now. It is under this scope that in 1944 John von Neumann and Oskar Morgenstern publish what was to be considered the founding base of game theory [5]. Even though their approach was evidently directed to economics, game theory soon showed to hold results of interest to other areas, such as political sciences, philosophy and biology. The field sure proved to be an interdisciplinary one,

where forces from all those fronts have given motivation and contributed to its development.

Essentially, game theory formalizes interactions between agents, by characterizing the complete set of possible actions that can be performed by them, and how those directly affect the outcomes each of them face. Classical game theory analysis assumes agents to have some level of rationality, in order to reach conclusions about their choices. On the other hand, evolutionary game theory does not rely on this assumption, as we will argue in section 2.3.

There are a plethora of games that have been extensively studied, such as the prisoner’s dilemma, the snowdrift game or coordination games, and an equally vast number of applications. Here we are interested in a particular class of them: signaling games (SG).

2.2.1 Signaling games

Due to the interactive nature of signaling, we can delve into its nuances resorting to a game-theoretic approach. This was initiated in 1969 by David Lewis [4], who proposed a new coordination game, that would later be coined the first signaling game. Two agents are present in each interaction of a general signaling game. One has some knowledge about the world, that may choose to inform or not the other about; while the other has enough agency to act on the world. The outcomes for both agents are generally conditional on the state of the world and on the performed act, but can also depend on the signals sent.

Lewis signaling game (common interests)

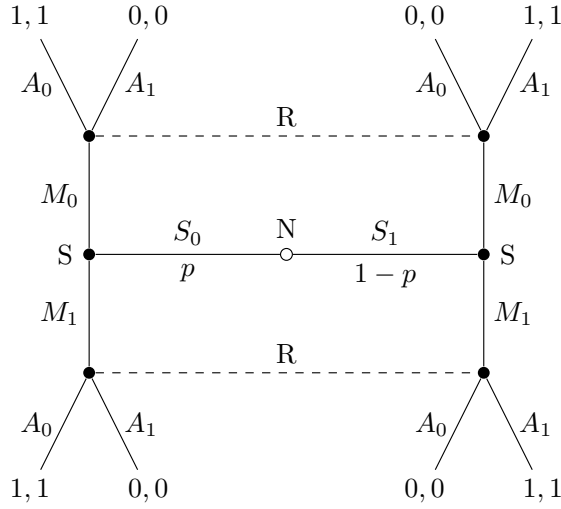
On this game, if the act performed matches the state of the world, both agents are rewarded, while contrary to that, they receive nothing. We proceed to formally define encounters under the Lewis signaling game, which are represented by its extensive form in fig. 2.1a.

We have the existence of a third agent, a fictitious one without strategic interests. This agent is commonly called nature (N) and is represented by a non-filled node. The game initiates with a move by nature (N), which chooses between states of the world S_0 and S_1 with respective probabilities p and $1 - p$. The sender (S) perceives the chosen state directly and depending on it, she will choose to send one of the two signals M_0 or M_1 . After observing the chosen signal and contingent on it, the receiver (R) will choose to perform one of the two acts A_0 or A_1 . The receiver does not have access to the original state chosen by nature. Hence the dashed lines connecting nodes from the same information set, which are indistinguishable to the receiver.

When nature chooses S_0 , act A_0 carries a payoff of 1 for both the sender and receiver, while act A_1 carries 0 for both. When nature chooses S_1 , act A_1 carries 1 for both, while act A_0 carries 0 both. This allows us to restate that there is a correspondence between states of the world and acts performed. When successfully matched, these bring a rewarding outcome to both agents. Because the payoffs depend only on those two variables, we represent the received payoffs as they are in fig. 2.1b.

Some of the more relevant features of this game are summarized here:

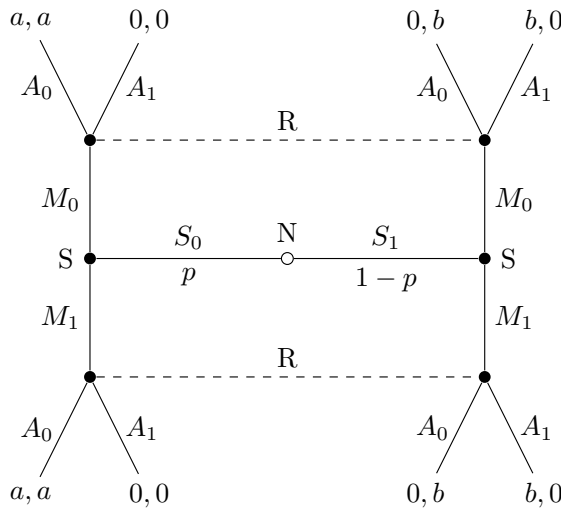
- Two-person – encounters are defined as being between two agents;



(a) Lewis game in extensive form.

	S_0	S_1
A_0	$(1, 1)$	$(0, 0)$
A_1	$(0, 0)$	$(1, 1)$

(b) Payoffs under Lewis signaling game.



	S_0	S_1
A_0	(a, a)	$(0, b)$
A_1	$(0, 0)$	$(b, 0)$

(d) Payoffs under signaling game with partially conflicting interests.

(c) Signaling game with partially conflicting interests in extensive form.

Figure 2.1: Two signaling games. The two games are equivalent in every aspect except for the received payoffs. The payoffs are first the receiver's, and second the sender's. The first move is a move by nature (N). It chooses between states of the world S_0 and S_1 with probabilities p and $1 - p$ respectively. Depending on the chosen state, to which the sender (S) has direct access, this agent chooses to send either signal M_0 or M_1 . The receiver (R) receives the chosen signal and, contingent on it, performs one of the acts A_0 or A_1 , without being directly aware of the state of the world. Hence the dashed lines, connecting nodes of equal perception by the receiver. Under the Lewis signaling game (figs. 2.1a and 2.1b), agents have aligned interests on both states of the world. Under the signaling game with partially conflicting interests (figs. 2.1c and 2.1d), this is no longer true: the agents have aligned interests if S_0 was chosen by nature, and conflicting interests if alternatively S_1 was chosen. In both cases, the signal chosen by sender does not affect directly the payoffs obtained by the two agents. This allows the payoffs to be represented depending only on the state of the world and act chosen (figs. 2.1b and 2.1d).

- Asymmetric – sender and receiver have different possible actions to choose from and as we will see, different strategic sets;
- Sequential – agents make their choices sequentially, first the sender and then the receiver;
- Perfect information – agents have access to the other agent’s choices if they were carried out before them (move by nature is not accounted, due to its fictitious character);
- Incomplete information – not all agents know about the exact outcome of each of their choices, since the move by nature is not accessible to the receiver;
- Two states, two signals, two acts;
- Pure common interests – sender and receiver always receive the exact same payoff on each encounter, and therefore should have the same preference over the acts to be performed.

Before expanding our understanding of signaling games, let us look into a possible altered version of the Lewis one.

Signaling game with partially conflicting interests

In section 2.1, we have asserted the empirical and theoretical motivation to explore signaling under partial conflict. Let us then relax the pure common interests present under the Lewis SG.

There are numerous formulations taking into consideration the existence of conflict under SG, some from specific economic systems [27–29, 48], other from biological ones [26, 31, 34], but we have opted to exhibit here a particularly general one [33, 41]. The game can be seen in its extensive form in fig. 2.1c, and because the new payoffs still depend only on the state chosen by nature and the undertaken act, we can exhibit them in the representation present in fig. 2.1d.

This game can be interpreted as a version of the Lewis one with two adjustments. The payoffs were generalized to variables a and b , depending on the state of the world, and the ones received under state S_1 were switched in a way that interests become opposed. This way, when the world is in state S_0 , both agents still prefer act A_0 to be performed; while when the world is in state S_1 , the sender holds that preference and the receiver prefers to perform A_1 . This should create a clear conflict: while the sender should prefer the receiver to act A_0 all the time, the receiver should prefer to receive information from the sender by means of signaling and match the two states of the world with the two respective acts.

2.2.2 Strategies and Information

Having introduced the main aspects of interactions under SG, it is worth examining the ways in which agents can proceed. The concepts introduced under this section are suitable for both the SG defined on the previous one.

Since each agent has to choose between a limited amount of signals – if we are considering a sender – or acts – if a receiver – we can define for each agent a correspondence between what she knows and the probability of choosing any of her prospects. This conditional decision process is what we call a strategy.

If we are dealing with pure strategies, as we will throughout this work, the probability of choosing one of the available possibilities will be 1, and 0 for the others. This means the decision will be deterministic given the information got by the agent. Looking at the sender, this would mean each state of the world having one unique link connecting it to a signal. Regarding the receiver, a strategy can be defined as each received signal having one unique link connecting it to one of the available acts.

Based on this definition, we can see that for N_S states of the world, N_M signals and N_A acts, senders will have $N_M^{N_S}$ possible independent strategies, while receivers will have $N_A^{N_M}$. If $N_S = N_M = N_A = 2$, as we have previously considered, each agent would have 4 possible strategies.

Information and signaling systems

With this definition of strategy in mind, we should consider what would a signaling system (SS) be under these circumstances.

In fig. 2.2, we can observe the representation of two of the four possible strategies a sender can have under a 2 state, 2 signal, 2 act game. There, each state of the world requires a correspondence to one signal. The game does not require the sent signals to be different, which means that both states of the world may be connected to the same signal, as happens in fig. 2.2a. In that case, it is clear that signals do not carry information about the state of the world.¹ This makes us say that they are uninformative.

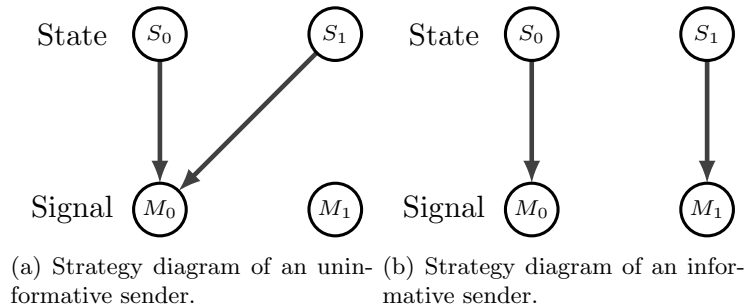


Figure 2.2: Sender strategy diagrams under the 2 state, 2 signal, 2 act signaling game. For each state of the world occurring, the sender has to choose the signal to be sent. This links each state to one unique signal. If the states are linked to the same signal (fig. 2.2a), the sender is uninformative. If the states are linked to different signals (fig. 2.2b), the sender is informative.

On the other hand, if we are looking at a situation where the sender discriminates between the two states of the world with different signals, as in fig. 2.2b, each signal carries 1 bit of information.² This means that signals change the probabilities of the state of the world, therefore carrying information, in fact the maximum amount of it. In these scenarios, we may say that senders are informative.

¹We can evaluate this quantitatively by introducing the concept of quantity of information contained in a *signal*, about the *states* of the world $I_{states}(signal)$. This definition comes from the Kullback-Leibler distance [69, 70] and is summed up in [1, Ch. 3, p. 36]. Quantity of information serves as a measure of the information gained when a *signal* is sent, or the distance between the initial probability distribution of the states of the world $-P(state_i)$ – and that distribution knowing that a *signal* was sent $-P(state_i|signal)$. If the logarithm function used is chosen to be base 2, then the quantity of information comes in bits.

$$I_{states}(signal) = \sum_i P(state_i|signal) \times \log \left[\frac{P(state_i|signal)}{P(state_i)} \right]$$

²This is true under the case where states are equiprobable. Otherwise, the quantity of information transferred would still be the maximum possible value under the circumstances, but always less than 1. See footnote 1.

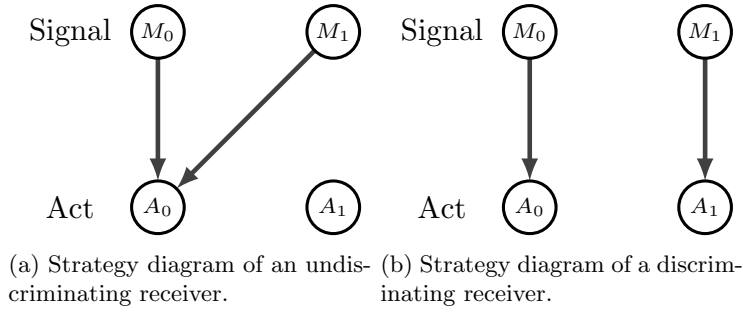


Figure 2.3: Receiver strategy diagrams under the 2 state, 2 signal, 2 act signaling game. For each received signal, the receiver has to choose the act to be performed. This links each signal to one unique act. If the signals are linked to the same acts (fig. 2.3a), the receiver is undiscriminating. If the signals are linked to different acts (fig. 2.3b), the sender is discriminating.

Moving on to consider receivers, in fig. 2.3 we have the representation of two of their possible strategies under the same previous setting. We see that each signal received has a correspondence to one act to be performed. Receivers may always perform the same act, as we see in fig. 2.3a, in which case they are undiscriminating. Or alternatively, they may perform different acts depending on the signal received, making them discriminating receivers, as the one we see in fig. 2.3b.

Even though the signaling game is symmetric in regard to switching signals M_0 and M_1 , the same cannot be said about the acts. There is an association between state of the world S_0 and act A_0 , and the same thing between S_1 and A_1 . This should be clear under the Lewis signaling system, since those are the acts that revert non-null payoffs to both agents. By interpreting the signaling game with partially conflicting interests as a modification of the first, we hold to that pairing of states and acts.

We can finally properly define signaling systems. These should be systems where senders are informative and receivers discriminate in a way that match each state of the world with the adequate act. The only two systems that fit this definition are the ones observed in fig. 2.4.

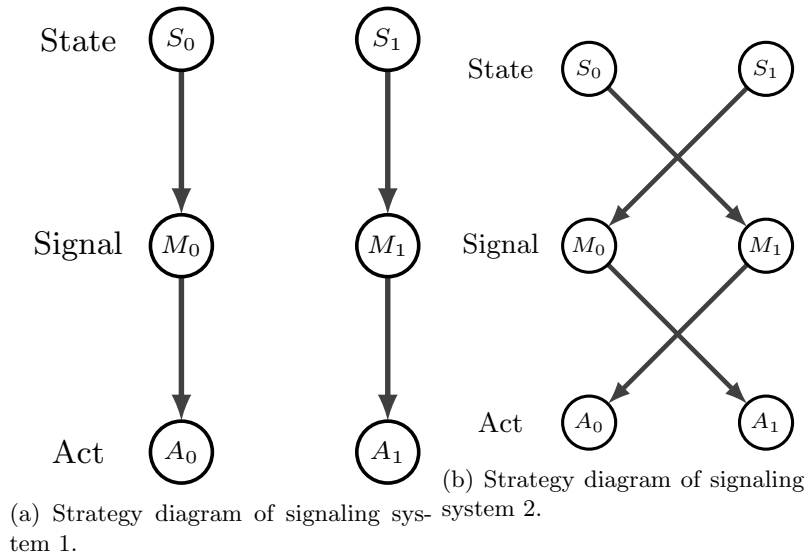


Figure 2.4: Combined sender-receiver strategy diagrams under the 2 state, 2 signal, 2 act signaling game. Signaling systems are defined state of the world S_0 being linked to act A_0 and state S_1 to act A_1 . This happens under the two combined strategies represented in figs. 2.4a and 2.4b.

Symmetrization

As we have stated, one of the characteristic features of the two SG here introduced was their asymmetry. The two agents have different sets of actions in the game and therefore different sets of strategies. For a variety of reasons, it may be useful to conceive a symmetric version of signaling games [71]. Under the present work, this will be particularly relevant due to the analytical tractability it brings to the analysis.

Under a symmetric version of the signaling game, each agent has an equal probability of being either the sender or the receiver on each encounter [71]. The expected payoff of one agent interacting with another becomes the average value of the payoffs obtained under the two roles. This changes the character of the model from being two-population into one-population, both previously shown to hold similar results under the Lewis signaling game [15, 19].

Because every agent becomes both a sender and receiver, this turns their strategies into a combination of their individual strategy as each of those two types. Therefore, each agent has a set of $N_M^{N_S} \times N_A^{N_M}$ possible independent strategies. If we consider, as we have done before, that $N_S = N_M = N_A = 2$, we get a total number of 16 strategies.

Strategies

Let us give a full characterization of all possible strategies, under the symmetric version of the signaling game. These 16 strategies are defined in table 2.1. Their representation was obtained the following way: {signal sent when state of the world is S_0 ; signal sent when state of the world is S_1 ; act performed when signal received is M_0 ; act performed when signal received is M_1 }. Additionally, we have indexed each of the 16 strategies based on the number their representation held under the binary numeral system (table 2.1). We have included the introduced definition of signaling systems (fig. 2.4) and anticipated the one provided for deceivers in the following section.

2.2.3 The strategic conflict

As we have stated from the beginning, we are interested in studying signaling systems and their origins. Under the interaction settings dictated by symmetric signaling games, are these systems expected to occur? Having defined these interactions under two signaling games in section 2.2.1 and explored some features of the agents' strategic possibilities in section 2.2.2, we are in a position to give a preliminary answer to this question.

Starting from the interaction framework introduced by the symmetric Lewis SG, we are able to compute the payoffs that signaling systems sustain against every other strategy. It should be clear that under a setting where two agents are using the same signaling system, they receive the maximum payoff 1 on every interaction, thus collecting the maximum possible average payoff 1. None of them is able to increase the received payoff by unilaterally changing to a different strategy. Because of this, we can say that signaling systems are likely to be adopted by rational agents looking to maximize their payoff.

Now moving to interactions under the symmetric SG with partially conflict interests, there is a set of two other strategies that becomes relevant, which we will call the deceivers (see strategies 1 and 14 in

Index	Representation	Name
0	{0, 0, 0, 0}	Deceiver 1
1	{0, 0, 0, 1}	
2	{0, 0, 1, 0}	
3	{0, 0, 1, 1}	
4	{0, 1, 0, 0}	Signaling System 1
5	{0, 1, 0, 1}	
6	{0, 1, 1, 0}	
7	{0, 1, 1, 1}	
8	{1, 0, 0, 0}	Signaling System 2
9	{1, 0, 0, 1}	
10	{1, 0, 1, 0}	
11	{1, 0, 1, 1}	
12	{1, 1, 0, 0}	Deceiver 2
13	{1, 1, 0, 1}	
14	{1, 1, 1, 0}	
15	{1, 1, 1, 1}	

Table 2.1: Numbered strategies and their representation. Deceivers are shown in brown and signaling systems in blue. Their representation was obtained the following way: {signal sent when state of the world is S_0 ; signal sent when state of the world is S_1 ; act performed when signal received is M_0 ; act performed when signal received is M_1 }. Additionally, strategies are separated by horizontal lines according to the signals sent (first two digits of their representation), which will later prove to be useful.

table 2.1). These should be interpreted in relation to SS. They are deceivers because as receivers they match the acts with the states of the world, but as senders they are uninformative. It is worth noting that they are not intrinsically deceivers, since there is no established agreement between agents. Deceiving, just as signaling, is interpreted here as developing behavior.

Let us look at fig. 2.5 where we exhibit the payoffs received by the signaling systems (SS) and their respective deceivers (D) under the mentioned setting with equiprobable states ($p = 0.5$). For finite non-zero values of a and b , it is evident that SS can increase their average received payoff by unilaterally becoming deceivers.

$$\begin{array}{cc}
 & \begin{array}{cc} SS & D \end{array} \\
 \begin{array}{c} SS \\ D \end{array} & \begin{bmatrix} \frac{2a+b}{4} & \frac{2a}{4} \\ \frac{2a+2b}{4} & \frac{2a+b}{4} \end{bmatrix}
 \end{array}$$

Figure 2.5: Payoffs obtained by signaling systems (SS) and their deceivers (D) when interacting under the signaling game with partially conflicting interests from fig. 2.1c. The payoffs exhibited are the ones received by the strategy in that row against the one in that column.

Here we have treated this problem in a rather simple and informal way. Nonetheless, it was enough to reveal how the notion of conflict under partially conflicting interests expands to the strategic space of the symmetric game. Why should we expect agents with enough rationality to signal under partially conflicting interests, if it is clear that deceiving against signalers guarantees higher payoffs? And how come we see that signaling systems ubiquitously coexist with partial conflict? To answer these questions we have to go beyond the superficial analysis exhibited here.

2.3 Evolutionary Dynamics

We could certainly be tempted to consider it enough that agents are benefited by deceiving to state that signaling systems have no means of emerging under conflict. However, we are not ready to abandon language to inexistence so fast and will try to reveal what may explain these systems resorting to evolutionary theory and population dynamics.

2.3.1 Evolutionary Theory

Evolutionary processes have been present in the scientific literature for some time, generally proposed in opposition to stationary accounts of the state of systems. In 1859 Charles Darwin publishes “On the Origin of Species” [72], a book that settled the foundation of evolutionary theory and would come to change the way we view complex systems on several levels.

The one process he thoroughly outlined in that book was natural selection. This process supposes three factors to be present: 1) natural variation, 2) differential reproduction, and 3) inheritance. When these three align as they do in biological systems, populations of agents go under evolution and they develop in the most fascinating ways. In that context, agents guarantee the inheritance of their characteristics by generating offspring. This inheritance is not done without mutations, which ensure the necessary natural variation. Finally, those characteristics have an impact on the agent’s reproductive chances, possibly by affecting its ability to survive or procreate. This guarantees the occurrence of differential reproduction based on the effect of the characteristics on the agent’s fitness.

Those three factors are not exclusive to the biological world. We can also find them in cultural evolution [73]. In some settings, self-interested agents imitate each other and this imitation process tends to favor the most successful of the available options. Their success is not measured here by the amount of offspring they leave, but by some characteristic specific to each setting that makes those options more desirable to the agents. Besides this, they explore and try different possibilities, even if these do not always guarantee the best outcome.

In 1868, Darwin publishes “The Variation of Plants and Animals under Domestication” [74], where the impact of an extra feature on evolutionary contexts is explored. In contrast with the work published 10 years before, here Darwin did not assume the staticity of the environment of which species are a part. Because each agent’s fitness may depend on the environment, its account should become dynamic and time-dependent.

This idea seems particularly interesting when the characteristics which are subject to evolution are related to the agents’ behavior. The actions of one agent should affect the fitness of all other agents in the population, and not just herself. This may happen both through natural or social selection. From that idea, and after Hamilton in 1964 [30], John Maynard Smith and George R. Price published in 1973 [9] what is considered the first analysis of interactive games under an evolutionary process. The advances done throughout the 70’s merging classical game theory from economics and evolutionary theory from biology culminated in the publication of the “Evolution and the Theory of Games” [10], which established the field of evolutionary game theory (EGT).

2.3.2 Signaling under EGT

Evolutionary game theory has been a fundamental framework across several fields ever since its development. It provides essential tools for the comprehension of phenomena in complex systems, without requiring any assumption about the rational faculties of the agents present in them. Under its lens, behavior such as altruism, cooperation, and signaling, may emerge in an evolutionary population. In this work, we intend to use this framework to better understand the evolutionary origins of signaling and its relation to conflict.

Several evolutionary models can be at use on game-theoretic approaches, but none has had as much attention as the **replicator equation** [7]. Here, populations are considered infinite and composed of agents with different strategies. The evolution of the relative frequency of each one of the strategies is then computed through the differential equation that names the model. This equation takes into consideration differential reproduction, but to study the additional effect of mutations, the replicator-mutator equation [8] should be used instead.

Some of the advantages of using the replicator equation are its tractability, as well as the simplicity of the analysis of equilibria through concepts such as evolutionarily stable strategy (ESS) [9, 10]. However, infinite population models may not be adequate for many reasons.

One of them is the fact that they are deterministic, therefore not taking into consideration the stochasticity to which most populations are exposed. On these grounds, **finite population dynamics** can reveal other facets of emerging phenomena [47]. These methods have been used considerably less to approach signaling and its origins. The approaches done up until now are rather isolated and either rely on pure common interests under the Lewis SG [17, 18], or rather specific accounts of conflict, such as a version of the battle of sexes [48] and the Sir Philip Sidney game [75].

In cases where the tractability of the problem is particularly difficult, agent-based models can be used. Under some of these, we can see the impact of individual learning being studied. Agents may have the ability to learn their preferred actions through reinforcement processes [76, 77], neural networks [78, 79], or update of beliefs [1, Ch. 8, p. 103–105]. Even though these may hold interesting results, they are not contemplated in the approach we intend on doing here.

Taking this into consideration, we state the need for an analytically tractable and systematic approach to signaling dynamics under conflict. Through its development, we hope to provide an analysis of the effects of stochasticity and finiteness in a population where partial conflict may exist. This was weighed in the designing of the model that will be presented in the following chapter 3.

2.4 Mechanisms for the emergence of signaling under conflict

The emergence of signaling in settings where agents have partially conflicting interests is uncertain. The problem is first explicit in the conflicting preferred acts to each agent (section 2.2.1). Then it assumes its form on a strategic conflict (section 2.2.3), which will certainly have an impact on the evolutionary dynamics of the system, and affect the emergence of signaling. However, we know from observation that meaning and signaling occur in such systems. In order to understand their evolutionary origins, several

mechanisms have been considered.

A large share of the models on signaling under conflicting interests rely on costs for signaling. One of the main motivations for this sort of mechanisms is Zahavi's handicap principle [22]. This principle says that for signals between animals to carry information, agents should incur in a cost when sending them. These costs could originate from factors such as energy spent or increased predation risk. They should translate into signals being sent only by senders to which they are affordable. In a milestone article by Grafen [23], we can see this idea being introduced for the first time in a solid way under signaling games. It consequently bridged the gap between the theoretical hypothesis and quantitative analysis. The mechanism has been studied to a great extent since then [24–29], its approach being usually based on defining a payoff matrix with conflicting interests and then parameterize the costs explicitly. Through these direct changes of the agents' interaction framework, signaling systems may effectively emerge.

Despite their thorough presence in signaling literature, the usage of costly signals to explain signaling emergence has been subject to some empirical criticism. Most of their critics argue that even though this framework's results appear consistent with what occurs in nature, most signals observed in natural systems are not costly enough [35–38]. This should be seen as an incentive to look for other mechanisms present in signaling systems.

Kinship is also seen as a potential signaling promoter. When agents are related, besides being interested in their own survival, they may also be interested in the survival of their kin. This should be strongly connected to protection of the genes they all share [30]. J. B. S. Haldane allegedly answered after being asked if he would risk his life for his brother's: "I will jump into the river to save two brothers or eight cousins" [80]. This illustrative example holds some mathematical value, by making evident the fraction of genes shared with our direct siblings ($1/2$) and cousins ($1/8$).

One of the main gateways to its introduction on quantitative models is through the idea of inclusive fitness, and the associated coefficient of genetic relatedness r [30]. The approach usually starts with the definition of an interaction matrix with the agent's individual interests, which is then followed by the computation of their fitness adding a contribution with the payoff of the other agent multiplied by r [26, 31, 32].

We should take into consideration that most models making use of inclusive fitness, such as the Sir Philip Sidney game, overlap that mechanism with costly signals, and by excluding those we may see signaling systems succumb [26]. Besides this, inclusive fitness has been subject to strong criticisms from the start due to the rigid assumptions it requires, which refrain it from being general enough [81].

Until now we have discussed the emergence of meaning and signaling in the cases where information transfer happens maximally, i.e. signaling systems. However, why should this be the only positive outcome? Individual agents, or populations in equilibrium, may spend a fraction of time signaling and acting accordingly and another fraction not doing it. This would be what we call non-maximal or partial information transfer. Some approaches to signaling study how partial information transfer can occur. For this to happen in our 2 state, 2 signal, 2 act game, we would have to resort to mixed strategies. Under this class of strategies, agents can adopt one of a set of strategies with defined probabilities on each encounter. If one of these mixed strategies were to be stable, we could call that a hybrid equilibria

of the system [31, 33].

In case this mixed strategy involved senders sometimes being informative and other times uninformative, that could be called a partial pooling equilibrium [32]. These equilibria may also occur with pure strategies under games with a larger number of states of the world. In fact, the resilience of partial pooling equilibria has been widely studied under N-state, N-signal, N-act games, for $N > 2$. That occurs when at least one state is properly signaled, while others are pooled.

Under costly signals, it is shown that partial pooling and hybrid populations/agents are often equilibria when signaling systems are not, because the costs required to sustain partial communication are proved to be lower than the ones required to sustain complete information transfer [31, 33]. Even though this is a more optimistic scenario compared to the classical formulation of handicap theory, costs are still essential to see meaning emerging on evolutionary populations.

The fact that agents may have repeated encounters between them has been studied as a feature possibly sustaining signaling under conflict [34]. Using an infinite population model, a signaling game is explored leaving receivers with the possibility of changing their strategy contingent on the one used by the sender. Paired up with conditional believers – i.e. receivers that act according to the signals received if the sender is honest but flee inescapably after being deceived – honest signalers are shown to be stable under one condition: the future benefits held by the sender are larger than the short-term benefit of lying. This result holds when signals are costless, which is extraordinarily refreshing. This shows that signaling systems under conflicting interests could be sustained by this adapted form of direct reciprocity.

Furthermore, population structure is a ubiquitously present feature of real complex systems that encompasses multiple possible underlying phenomena. These phenomena may include spatial configuration [79, 82, 83], population viscosity [84] or even discriminating partner choice with the most various origins (resemblance [85], reciprocity and mutualism [34, 86, 87], interest [55], etc.). All these confer correlated interactions between agents [88], thus breaking the well-mixed barrier and generating structured populations. In some cases, this is enough to promote the emergence of promising behavior, such as cooperation [52, 55, 56, 89–92]. However, there are not many theoretical approaches to its effect on signaling, therefore leaving a lot of questions open. Let us look at the ones done so far.

In the first part of [83], the effect of spatial structure on the classical 2 state, 2 signal, 2 act Lewis game is studied. This structure is represented by a 2D lattice under which agents interact and imitate their neighbors. This is seen to raise interesting aspects that are not considered in well-mixed populations, being the most interesting one of them regional meaning. Populations regularly collapse into states where every agent is in a signaling system, but not necessarily in the same one.

These results were extended in [93] to different topologies of static interaction networks going from regular rings to random graphs. It is concluded that small-world networks tend to facilitate the achievement of a universal signaling system in ambitious scenarios not covered by well-mixed models. These include the states of the world not being equiprobable and existing more than two states. One of the main interests of this result is the pervasive character of small-world networks and the almost certain positive outcome of communication's attempts under them. However, these results are held under the no-conflict assumption inherent to Lewis SG.

A more general approach to signaling dynamics under deterministic population structure is done in [41], making use of adaptive interaction networks [55]. Adaptive networks have been thoroughly explored in the past decades, due to their pervasiveness across all sorts of interactive complex systems [42–46]. Signaling systems should not be an exception. The used active linking model [55] automatically couples the dynamics of adaptive networks with the evolutionary dynamics of the population. It is concluded that signaling systems may be resistant to the fixation of deceivers under infinite populations if the links formed between the two types of strategies have high death rates. However, by solely studying the parameter space allusive to those links, this advance lacks more robust conclusions about the evolutionary stability of signaling under adaptive networks.

These models have only considered coinciding interests [83, 93] or truncated versions of the signaling game with a reduced set of 2 or 3 strategies, both being premises which may provide less restrictive conditions of signaling emergence [19]. In addition the only approach developed under conflict interests [41] uses infinite populations' notions, such as that of evolutionarily stable strategy, which allow deterministic outcomes and simple conclusions but ignore the stochasticity of systems. This stochasticity has been previously shown to hold different promising outcomes [47], as mentioned in section 2.3.2.

Taking this into consideration, we stand for the interest to reveal the role of population structure and adaptive interaction networks on evolutionary populations and understand how their dynamical coupling impacts the emergence of signaling systems. We wish to implement this under a finite population model that accounts for the stochastic character of behavior, hoping this will give us new insights into the occurring non-trivial dynamics. All these aspects of the model we will be using are thoroughly defined in chapter 3.

Chapter 3

Model

In this chapter, we outline the model to be adopted throughout our approach to signaling. We start by proposing a signaling game with a novel unifying payoff framework in section 3.1. Then in section 3.2, we will build the foundation of the stochastic dynamics and go through the computations required to assess signaling prevalence. In section 3.3, we will proceed to the introduction of adaptive interaction networks through a review of the active linking framework. This will pave the way to the original partner choice model designed in section 3.4. In the final section 3.5, we provide an overview of the parameters present in this model.

3.1 Conflict in Lewis signaling game

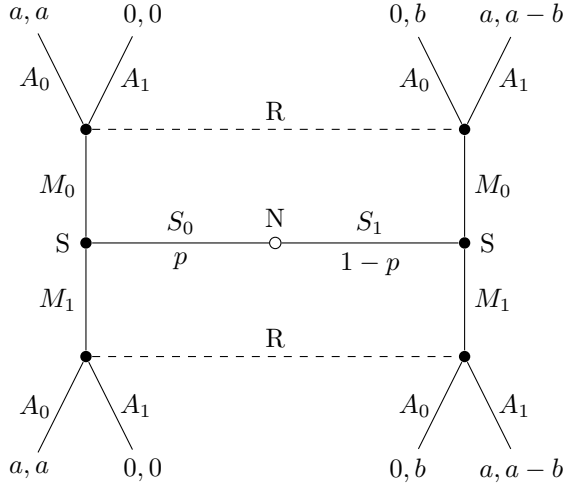
We propose the game defined by its extensive form in fig. 3.1a. Following section 2.2.2, we will be using the symmetric version of this game in our model. This formulation introduces a continuous model with two parameters¹, that in one of its limits generates the Lewis game [4] and on the other a version of the partially conflicting interest game defined in [33] – both games previously introduced in section 2.2.1. It can also be seen as a formulation of the Lewis signaling game with gaugeable partial conflict.

Parameter a of this game should be interpreted as the reward of successful interactions between the two agents. In the original Lewis game [4] and most of its subsequent analysis, this parameter is defined as 1. Parameter b is introduced to include some level of partial conflict. We keep saying partial conflict, because in state S_0 , interests align completely, so conflict is only possible when in state S_1 .

This formulation is one of the original contributions of this work to the mathematics of signaling games. It allows one to gauge between different levels of conflict b/a . When $b/a = 1/2$, the sender has no preference over the executed acts. For values $b/a < 1/2$ there is no partial conflict and in the limiting scenario $b/a = 0$ this game is equivalent to that of Lewis. On the opposite $b/a > 1/2$, we have partial conflict, attaining its maximum level for $b/a = 1$, where this game becomes equivalent to a specification of the game in [33, 41].

¹This payoff framework uses parameters similar to the ones explicit in section 2.2.1, which in turn were inspired from [33]. However, their use and meaning do not coincide with those of that section.

²See footnote 1.



(a) Proposed signaling game in extensive form.

	S_0	S_1
A_0	(a, a)	$(0, b)$
A_1	$(0, 0)$	$(a, a - b)$

(b) Payoffs of proposed signaling game.

Figure 3.1: Signaling game with the proposed unifying payoff framework. Its extensive form is in fig. 3.1a. Its payoffs are summarized in fig. 3.1b. The payoffs are first the receiver's, and second the sender's. Except for the payoffs, this game is the same as the ones elaborated in fig. 2.1². For a detailed description of the extensive form, see fig. 2.1. When $b = 0$, the game becomes the Lewis signaling game (figs. 2.1a and 2.1b). When $b = a$, the game turns into a canonical signaling game of partially conflicting interests (figs. 2.1c and 2.1d). Strategies under the symmetric version of this game are listed in table 2.1.

We exhibit a simplified representation of its payoffs in fig. 3.1b, which depend only on the state of the world and the performed act. Again, the first value is the payoff received by the receiver in the encounter, and the second is the one received by the sender.

3.2 Stochastic evolutionary dynamics

In a population of Z agents, two of them are randomly chosen. Let us call one of them agent i and the other agent j . Each of these agents will then interact with all other $Z - 1$ agents in the population, half the times as senders and the other half as receivers. It is worth noting that nature can chose either state S_0 or S_1 with probabilities p and $1 - p$ respectively. That way, the average payoff received by an agent with strategy A when interacting with an agent with strategy B is the following:

$$\pi_{AB} = p \cdot \frac{\pi_{A \rightarrow B}(S_0) + \pi_{A \leftarrow B}(S_0)}{2} + (1 - p) \cdot \frac{\pi_{A \rightarrow B}(S_1) + \pi_{A \leftarrow B}(S_1)}{2} \quad (3.1)$$

where $\pi_{A \rightarrow B}(S_i)$ is the payoff received by agent with strategy A , when interacting as sender with an agent with strategy B and $\pi_{A \leftarrow B}(S_i)$ is the payoff received by agent with strategy A , when interacting as receiver with an agent with strategy B , both under state of the world S_i . The payoffs are defined by the simple matrix from fig. 3.1b, and in reference to the game represented in its extensive form in fig. 3.1a.

In order to obtain the fitness Π_i and Π_j of the two initially chosen agents, we compute their average payoff against all the other $Z - 1$ agents in the population, using eq. 3.1.

$$\Pi_i = \frac{1}{Z - 1} \sum_{k=1, k \neq i}^Z \pi_{s(i)s(k)} \quad (3.2)$$

where $s(i)$ and $s(k)$ are, respectively, the strategies of agents i and k in the population. As a result, the computation of the value of their fitness should depend only on their strategy and the relative frequency of each strategy on the population.

Based on the two chosen agents' fitness, we will apply what is usually called the pairwise comparison [11, 49–51, 94], where the Fermi function is used as a stochastic update rule (equation 3.3). This will define the probability of the first agent i changing from her current strategy $s(i)$ into the one the second agent j is using $s(j)$, based on their just computed fitness.

$$P_i(s(i) \rightarrow s(j)) = \frac{1}{1 + e^{-\beta(\Pi_j - \Pi_i)}} \quad (3.3)$$

In the symmetric signaling game being used here there is no separation between the sub-strategy the agent uses as a sender and the one used as a receiver. When the imitation process occurs, strategy $s(j)$ is entirely imitated. See the possible strategies and their indexation stated in table 2.1.

Other evolutionary methods could be used here, but we have chosen to follow the work done in recent years regarding the use of the Fermi function from statistical physics. This account provides an effective parameterization of the intensity of selection as the temperature β . The intensity of selection serves as an attribute of an evolutionary system, that indicates how sensitive the process is to differences in the fitness. We can interpret it as accounting for the intensity of noise in the imitation process [6, 82], which means that lower β will bear higher dispersion around the best decision.

Moreover, β allows us to have a continuous model between two interesting limiting evolutionary processes. On one side $\beta \rightarrow 0$ gives us probability of transition $p = 1/2$, which means that the population will be under neutral drift. On the other side of the scale, for $\beta \rightarrow \infty$, we will have the deterministic imitation dynamics happening: agent i will certainly change her strategy to $s(j)$, if and only if $\Pi_i < \Pi_j$.

We introduce at this point the mutation probability μ of agents choosing any of the other strategies equiprobably, even if some are not present in the population. When one agent i from the population is chosen to possibly change her strategy, there are two scenarios: with probability μ , she can choose any of the other possible strategies equiprobably, i.e. mutate; or with probability $1 - \mu$, she can undergo the just described pairwise comparison. This introduces more randomness into the evolutionary model and reproduces the effect of mutation, an essential feature in evolutionary theory.

Through this randomness, mutations leave the model used here less analytically tractable, which makes us take into consideration the small mutation limit $\mu \rightarrow 0$ [12]. Under this limit, the evolutionary process should consist on consecutive pairwise comparisons with very rare mutations, therefore leaving the population tending to one of the states where every agent in the population uses the same strategy, which we will call monomorphic states. The population then freezes in the first monomorphic state it reaches, waiting for a mutation to introduce an intruder. When this happens, there are two possible scenarios emerging through pairwise comparisons: (1) the mutant strategy may spread on the population, eventually fixating and overthrowing the previously present strategy; (2) the settled population will extinct the new mutant strategy, and it will return to be what it was before the mutation happened. It is evident that in both cases, the system should reach another monomorphic state and stay there for a

comparatively long time before a new mutation happens. This allows us to consider the evolutionary process to be a population moving between monomorphic states.

For this approximation to be accurate, it should be confirmed that all the polymorphic states are indeed transient in the non-approximated process [12], otherwise the range of mutation probabilities for which the obtained results hold could be constrainingly small [53]. Under the Lewis Signaling Game ($b = 0, p = 0.5$) all non-monomorphic rest points have been shown to be unstable [15, 19]. In case this does not hold under non-null conflict levels ($b > 0$), the approximation could be extended considering further configurations of interest apart from the monomorphic states, that way achieving more accurate results [53].

In order to formalize this setting, we follow [12, 48, 50, 51, 94] and consider an approximate discrete-time Markov chain with the n_s monomorphic states, $n_s = 16$ being the number of possible strategies (see table 2.1). The transition probability between monomorphic states with strategies A and B is the probability of one agent with strategy A mutating to strategy B , times the probability (under the pairwise comparison process) of one mutant using B to **fixate** on a population of agents using A .

To compute the **fixation probabilities**, we start by asserting that in a population with $Z - k$ agents with strategy A and k mutant agents with strategy B , the probability of having the number mutants increase (+) or decrease (-) by one is

$$T^\pm(k) = \frac{k}{Z} \frac{Z - k}{Z} \frac{1}{1 + e^{\mp\beta(\Pi_B(k) - \Pi_A(k))}} \quad (3.4)$$

The fitness $\Pi_A(k)$ and $\Pi_B(k)$ is written in function of the number of mutants k due to the fact that, under this model, transitional populations will consist only of two strategies and are therefore well defined by that number alone. Following the steps explicitly outlined in [95], we are able to attain the probability $\rho_{A,B}$ of having one mutating agent with strategy B fixating in a population of agents with strategy A (equation 3.5).

$$\rho_{A,B} = \left[\sum_{l=0}^{Z-1} \left(\prod_{k=1}^l \lambda_k \right) \right]^{-1} \quad (3.5)$$

$$\lambda_k = \frac{T^-(k)}{T^+(k)} = e^{-\beta(\Pi_B(k) - \Pi_A(k))} \quad (3.6)$$

The processes of fixation are an essential part of this model, since they will determine the population dynamics. Because this model has a stochastic nature, all these fixations are possible for finite values. This way, a good metric to evaluate if fixations are relevant or not, is comparing them to the ones occurring under the neutral drift. As already stated under the introduction of the pairwise comparison, the neutral drift occurs when we have a very weak intensity of selection $\beta \rightarrow 0$, or alternatively when the selected strategies obtain exactly the same fitness. Under this limit, $\lambda_k = 1$ and the probability of neutral fixations is therefore $\rho = 1/Z$ [11]. Here we introduce the concept of **evolutionarily robust strategy** (ERS) [13, 14]: one where all single mutants have fixation probabilities below neutral fixation. $1/Z$. These concepts will be essential on the dynamics' analysis to be performed.

Now we have enough to build the transition matrix $M_{n_s \times n_s}$ and define the Markov chain of this stochastic process. We use the values of $\rho_{A,B}$ in the entries that represent transitions between different monomorphic states, i.e. when $A \neq B$. An appropriate coefficient $\eta = (n_s - 1)^{-1}$ is introduced, giving a proper normalization of the mutation process. The remaining diagonal elements should be filled with the probability complementary to the row where they are located. This will make sure that the sum over each of the matrix's rows is equal to 1, a requisite for it to be a transition matrix. **All the matrix elements are numbered from 0 to 15, corresponding to indices from table 2.1.**

$$M = \begin{bmatrix} 1 - \eta(\rho_{0,1} + \dots + \rho_{0,n_s-1}) & \eta\rho_{0,1} & \dots & \eta\rho_{0,n_s-1} \\ \eta\rho_{1,0} & 1 - \eta(\rho_{1,0} + \rho_{1,2} + \dots + \rho_{1,n_s-1}) & \dots & \eta\rho_{1,n_s-1} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & 1 - \eta(\rho_{n_s-1,0} + \dots + \rho_{n_s-1,n_s-2}) \end{bmatrix} \quad (3.7)$$

Having the discrete-time Markov chain defined, we are capable of obtaining the stationary distribution φ . This corresponds to the distribution over the 16 monomorphic states, which will not be altered by the application of the transition matrix. This way, the stationary distribution corresponds to the left eigenvector with eigenvalue equal to 1 of this matrix [12].

$$\varphi = \varphi M \quad (3.8)$$

This distribution will be the key result of the analysis, since it will allow us to evaluate the prevalence of each of the monomorphic states. Since our main objective here will be studying signaling emergence, we define the following quantity σ as the signaling prevalence. **All the vector elements are numbered from 0 to 15, corresponding to indices from table 2.1.**

$$\sigma \equiv P(\text{signal}) = \varphi_5 + \varphi_{10} \quad (3.9)$$

Under the small mutation limit, transient states of the population are considered to be rare and the time spent in them negligible. This way, the signaling prevalence σ accounts not only for the prevalence of monomorphic states of signaling systems 5 and 10, but unavoidably for the prevalence of signaling in the overall dynamics.

3.3 Adaptive interaction networks

We have worked up until now under the assumption that the population at stake was well-mixed. This was made clear during the computation of the agents' fitness, where encounters with every other agent were considered to have the same weight. As pointed out in section 2.4, rejecting this assumption may provide original insights about signaling emergence.

Following this, we introduce adaptive interaction networks. Interaction networks, or graphs, can be

used to model populations, where agents are defined to be the nodes and the encounters between them occur according to their weighted links. This allows us to select which agents may interact with each other, the probabilities of those interactions and how they evolve in time. It subsequently introduces new population topologies and the establishment of a new dynamical process in the population – network dynamics – on top of the already existing one – strategy selection. From this feature, the term **co-evolution** [52, 54–56] arises.

In order to model the network dynamics, we introduce the **active linking** framework [41, 52, 54–56], which was particularly designed in the context of evolutionary game theory. Under this framework, agents using a strategy A have a propensity to form new links α_A , which can vary between 0 and 1. Links between agents using strategies A and B form at a rate $\alpha_A\alpha_B$. The lifetime of those links is τ_{AB} , which is the inverse of their death rate $\gamma_{AB} = \tau_{AB}^{-1}$. Having this in mind, we can write the differential equation for the number of active links X_{AB} between agents using strategies A and B

$$\dot{X}_{AB} = \alpha_A\alpha_B(N_{AB} - X_{AB}) - \gamma_{AB}X_{AB} \quad (3.10)$$

where N_{AB} is the total number of possible links between agents with those two strategies, which is $N_{AB} = N_A N_B$ when $A \neq B$. N_A is the number of agents using strategy A in the population. Equation 3.10 also applies for links between agents using the same strategy $B = A$, we just need to use the appropriate $N_{AA} = N_A(N_A - 1)/2$

The previously mentioned co-evolution can be taken into its limits [54, 55], with the objective of making this model analytically tractable. The limits are related to the typical time of the active linking dynamics τ_a compared to the typical time of the strategy selection process τ_s . In one extreme, we have $\tau_a \gg \tau_s$, where the network is approximately static, thus making the evolution of the population happen on its initial graph configuration [89], completely defined under this model by X_{AB}^0 . In the middle, we get $\tau_a \sim \tau_s$, where it is virtually impossible to compute the outcome without resorting to numerical simulations [55]. On the opposite side of the scale, we have $\tau_a \ll \tau_s$, where the network evolves much faster than the imitation dynamics, thus being in its equilibrium topology defined by X_{AB}^{eq} , at all times [52].

We are interested in studying the evolutionary process under adaptive networks using the assumption that $\tau_a \ll \tau_s$, as it is done in [52]. To do so, we should compute the equilibrium distribution X_{AB}^{eq} , which is obtained setting the time derivatives in the differential equation 3.10 to 0 (equation 3.11). This will give us the amount of active links between agents using A and B under our model at any moment.

$$X_{AB}^{eq} = \frac{\alpha_A\alpha_B}{\alpha_A\alpha_B + \gamma_{AB}} N_{AB} = \phi_{AB}^{eq} N_{AB} \quad (3.11)$$

How should these network dynamics affect our previous formulation of the population evolution? The difference introduced is that agents under these will not be linked to all other agents. To account for this, we should adjust the way we compute their fitness. Previously, we have considered Π_A to be the regular average over the payoff the agent received against every other agent in the population (eq. 3.12). Now, the altered fitness Π'_A takes into consideration that only a fraction ϕ_{AB}^{eq} of the original interactions

do occur (eq. 3.13). The Kronecker delta δ_{AB} in the equations is equal to 1 if $A = B$ and to 0 if $A \neq B$.

$$\Pi_A = \frac{1}{Z-1} \sum_B \pi_{AB}(N_B - \delta_{AB}) \quad (3.12)$$

$$\Pi'_A = \frac{1}{Z-1} \sum_B \pi_{AB} \phi_{AB}^{eq} (N_B - \delta_{AB}) \quad (3.13)$$

It is suggested from a comparison between the two equations, that the active linking framework ends up introducing a simple transformation of the strategic payoff matrix, where the effectively received payoffs π'_{AB} are

$$\pi'_{AB} = \pi_{AB} \phi_{AB}^{eq} \quad (3.14)$$

It is worth noting that by considering the limiting case where the AL's parameters are $\alpha = 1$ and $\gamma = 0$, all agents will be connected to each other and we get a static complete graph. We can see that it is rather easy to recover, under the AL framework, the well-mixed population as well as the dynamics developed previously to this section.

3.4 Partner choice norms

The active linking framework introduces several new parameters: one propensity value α for each strategy and one death rate γ for each combination of two strategies. These are the parameters which under adaptive interaction networks define the final structure of the population. Then what values should be chosen for these parameters?

When we are talking about the one-shot prisoner's dilemma [52, 54–56], there are only two possible strategies: to cooperate or to defect, which leaves the number of active linking parameters to be only 5. In [92], we can see AL applied to the repeated prisoner's dilemma and in [41] to our formulation of the signaling game, which are both cases where the number of analyzed strategies could be larger. The problems were however reduced to the direct conflict between two strategies, thus indirectly reducing the number of AL's parameters. Because in this work we will keep track of the dynamics between the whole 16 possible strategies, this model of adaptive interaction networks introduces a total of $16 + 16 \times (16 + 1) / 2 = 152$ parameters. This makes it virtually impossible for us to do a digestible analysis of the full parameter space as the ones done in [41, 55, 92], which cover all the population structures possible under the scope of active linking dynamics.

We propose here the concept of **partner choice norm**. The main aim of this formulation is to introduce population structure arising from agent's individual linking inclinations. Resorting to simple considerations, we start by defining the ways under which agents may have linking inclinations towards each other. The death rate values under the adaptive interaction network γ_{AB} are then obtained evaluating the inclination agents with strategies A and B have towards each other. If both agents are inclined towards each other, we attribute to that link a slow death rate γ_S ; if none is inclined, we attribute to it

a fast one γ_F ; and if their inclinations do not agree, we attribute the medium value $\gamma_M = (\gamma_S + \gamma_F)/2$. All propensities are left as $\alpha = 1$. The values of γ_S and γ_F are defined through parameter δ the following way:

$$\gamma_{F/S} = 0.5 \pm \delta \tag{3.15}$$

Parameter δ represents the **degree of discrimination** upon which all agents in a population will act when being inclined or not inclined to link. When this parameter is at its minimum $\delta = 0$, all agents will link the same way, independent of their inclinations. The fraction of active links will be the same between every two strategies $\phi^{eq} = 1/(1 + 0.5) = 2/3$ (from eq. 3.11). When at its maximum value $\delta = 0.5$, the effect of the agent's inclinations will be felt the strongest and the structure of the population will be the furthest from the well-mixed scenario.

Let us look into the partner choice norms which were chosen to be later studied in depth. These are just some of the numerous possibilities of existing linking dynamics, and they were chosen based on their supposed logical simplicity [96–98]. They generally assume agents to be able to acknowledge the actions performed by each other, and the payoffs received after their encounters. Additionally, agents are supposed to have repeated interactions, so that they are able to identify each other based on a memory of what occurred in previous encounters. We introduce two **signal-based norms** – Informative and Convention – under which being inclined depends only on the signals sent by the agents. These will be the main focus of this work. Additionally, we introduce one example of an **outcome-based norm**, under which partner choice is determined by the payoffs received when interacting with the other agents. Each of the norms leads to the definition of the death rate matrices displayed in the appendix A.

Informative norm (signal-based)

The Informative norm is a signal-based norm centered around choosing informative agents. By discriminating between the two states of world, those agents' signals carry information about it. This means that inversely, agents using pooling strategies will be avoided. To distinguish between these, agents interacting as receivers need to be able to infer the true state of the world from the payoff resulting from each encounter. This norm is in a sense universal, since all agents are inclined towards the same set of other agents.

Convention norm (signal-based)

Under this signal-based norm, agents are inclined to form links with other agents that use the same signals they use for each of the two states of the world. As this is another signal-based norm, it also requires that agents are able to perceive the payoff received when interacting with their peers and understanding from it the true state of the world.

However, contrary to the previous one, this norm is not universal, meaning that the strategies with which each agent is inclined to link, depend on her own strategy. Nonetheless, this strategy seems to have a different interesting feature, which is its reciprocity. If one agent is inclined to form a link with another, this second agent will also be inclined towards forming a link with the first. This feature will separate the set of strategies into cliques – i.e. a group of strategies which are all inclined to each other,

but not to any other strategy outside that group.

Outcome-based norm

Here we present one possible formulation of an outcome-based norm, relying on distinct information from the encounters. If the average payoff received by one agent in encounters with another agent is satisfactory, they are inclined to link. Otherwise, if the payoff is not satisfactory, they avoid that link. In order to be satisfying, the payoff should be at least equal to the one received by signaling systems under that setting: $a - (1 - p) \cdot \frac{b}{2}$. Contrary to the other introduced norms, the dynamics generated by this vary with the parameters of the payoff framework. In section A.2 we provide the three death rate matrices observed in distinct intervals of b/a , when $p = 0.5$.

Outcome-based linking has been previously explored under strategic games. In [99], outcome-based partner selection is shown to be evolutionarily stable in the context of a collective risk dilemma with pre-selection of partners. In [55], an informal version of outcome-based partner choice is used – the death rates of active links in a network are set at real values inversely proportional to the average received payoff. This model shows that cooperation can be promoted by the developed population structure.

3.5 Overview of model’s parameters

Parameter	Symbol	Range	Figure
Population Size	Z	{20, ..., 50 , ..., 250}	B.3
Intensity of Selection	β	{0.05, ..., 1 , ..., 200}	B.1, B.2
Game parameter a	a	{1}	
Game parameter b	b	{0, 0.1, ..., 1}	4.1, 4.4
Conflict Level	b/a	b/a	4.1, 4.4
Probability of state of the world S_0	p	{0, 0.05, ..., 0.5 , ..., 1}	4.1, B.4
Degree of Discrimination	δ	{0, 0.05, ..., 0.25 , ..., 0.5}	4.5, B.1, B.3
Linking Propensity	α	1	
Slow death rate	γ_S	$0.5 - \delta$	
Fast death rate	γ_F	$0.5 + \delta$	
Medium death rate	γ_M	0.5	

Table 3.1: Parameters present in the model and parameter space analyzed. The values highlighted in the range represent the standard ones being displayed in the next chapter.

Chapter 4

Evolution of Signaling in Adaptive Networks

In this chapter, we intend to exhibit and discuss the results obtained through the evolutionary model just developed. In section 4.1, by using the proposed unifying payoff framework, we assess the impact conflict has on the emergence of signaling. Focusing on the limiting case where partial conflict is maximum, in section 4.2 we provide an interpretation of the mechanisms underlying the population dynamics. In section 4.3, we drop the well-mixed population ground and probe how signal-based partner choice may affect signaling prevalence and the overall dynamics. In section 4.3.2, one outcome-based norm is explored as an alternative to the mechanisms on which we primarily focus. The chapter ends in section 4.4, with a review of all the conclusions drawn on it.

4.1 Signaling under different conflict levels

We have provided a rather new payoff framework that couples the concept of conflict to the well known Lewis signaling game. By means of this systematic approach to conflict, this picture provides a fresh new insight on how we may see its impact on signaling emergence. Under this unifying framework, the ratio b/a is interpreted as the conflict level. When there is no conflict $b/a = 0$, it produces the Lewis signaling game, while when conflict is maximum $b/a = 1$, we see one of the possible games described in [33, 41]. It allows us to link results obtained under the Lewis game [17, 18] to the ones under conflict scenarios parallel to [48, 75]. The signaling prevalence σ obtained for different conflict levels b/a , varying b , is shown in fig. 4.1.

First focusing on the $p = 0.5$ curve, the Lewis game limit $b/a = 0$ holds signaling system prevalence of 99.996%. This result is as promising as the ones exhibited in [17, 18], when considering the same game setting with 2 states, 2 signals and 2 acts, under the frequency-dependent Moran process. The signaling prevalence holds values $\sigma > 99\%$ for conflict levels $b/a \leq 0.3$, therefore exhibiting a plateau where the dynamics observed under the Lewis game should hold. Above that value, signaling prevalence drops considerably, attaining the minimum value of $\sigma = 12.9\%$ for $b/a = 1$. We can get σ as high as 23%

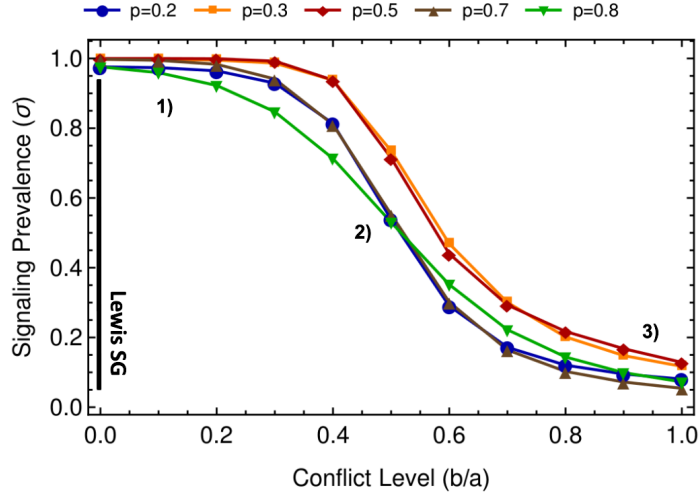


Figure 4.1: Signaling prevalence under changing values of conflict level b/a and probability p of state of the world S_0 . Three signaling regimes are marked: 1) dominance, 2) prevalence and 3) relevance. The different values of the conflict level b/a were obtained indirectly through the variation of parameter b . On limit $b/a = 0$, this setting becomes the Lewis signaling game; on limit $b/a = 1$, the level of partial conflict becomes maximum. Other parameters: $Z = 50$, $\beta = 1$, $a = 1$.

under this limit, if we stick to lower values of β , as it can be seen under the tests for robustness. These values seem to be coherent with the results in [48], where signaling prevalence stays between 15% and 25%, for similar values of Z , p and the payoff matrix. Even though this last result may not seem very promising, it is the worst-case scenario concerning conflict and we are still able to see signaling occurring a relevant fraction of the time. This comes to show that signaling can be at least evolutionarily relevant at any conflict level, a result impossible to obtain under deterministic models of infinite population.

It is particularly interesting to think about the way increasing the conflict level b/a affects the game. When changing from $b/a = 0$ to $b/a = 0.5$, the best act A_1 under S_1 progressively stop representing that much of a difference to the payoff the sender gets. In fact, at $b/a = 0.5$ the sender is indifferent to which of the acts is used. This is where the game shifts to a partially conflicting interests one. Therefore, we can connect this information directly to the three regimes exhibited in fig. 4.1:

1. Complete **dominance** over other states – signaling has a prevalence of over 99% – seen under strong alignment of interests;
2. **Prevalence** with the relevant occurrence of other states – signaling is the most prevalent strategy, even though prevalence values are lower than 99% – seen under weak alignment and weak partial conflict of interests;
3. **Relevance** under the prevalence of deceiving (as will be seen in detail in the next section) – signaling is not the most common strategy, but it has a prevalence of over 5% – seen under strong partial conflict of interests.

Now looking into the other curves in fig. 4.1, we state that large deviations from the equiprobable setting $p = 0.5$ only lead to a small decrease in signaling prevalence σ . Deviations from the equiprobable scenario ($p \neq 0.5$) are pointed out as an obstacle to the emergence of signaling systems under the Lewis

signaling game in [1, Ch. 5, p. 66], supported by the results obtained with the two-population replicator-mutator model in [19]. Under finite populations, we see that the picture appears more promising, where for the interval $0.2 \leq p \leq 0.8$ signaling prevalence under the Lewis game is always $\sigma > 97.6\%$. Besides this, all values of p exhibit curves with approximately the same shape, meaning that the 3-regimes picture shown in fig. 4.1 is robust under large p deviations.

4.2 Evolutionary dynamics under maximum partial conflict

We will now move to a deeper understanding of the evolutionary dynamics underlying the worst-case scenario where partial conflict is maximum $b/a = 1$. The results were obtained still under a well-mixed population and can be understood by looking at the graph with transitions between monomorphic states (fig. 4.2), and the resulting stationary distribution (fig. 4.3). This last will tell us the prevalence of each of the 16 possible monomorphic states which, under the small mutation limit, should be equivalent to the prevalence of each of the corresponding 16 strategies. Before getting into the analysis itself, we advise going back to table 2.1 in order to understand what the numbered strategies mean and what are the names used for each strategy.

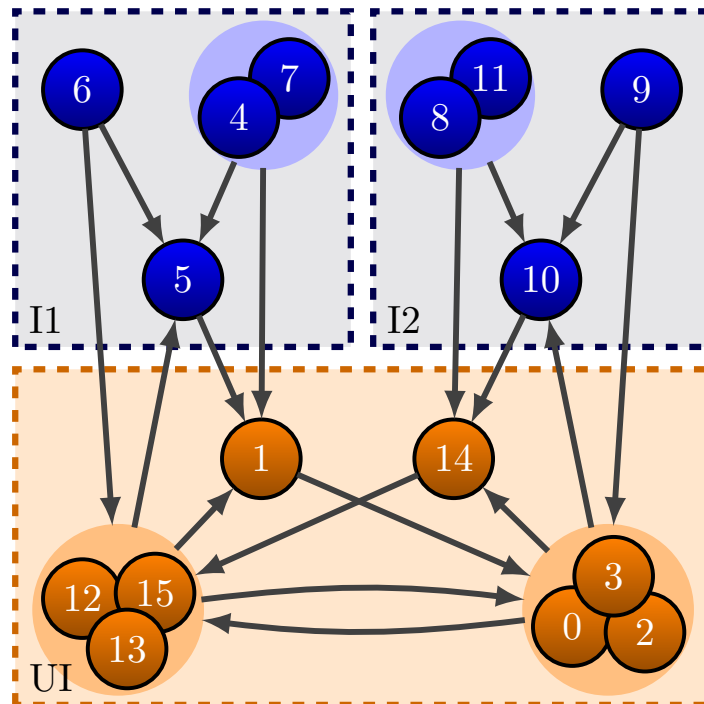


Figure 4.2: Graph of the resulting dynamics between the 16 monomorphic states under maximum partial conflict. Because of the stochastic nature of the evolutionary dynamics, all transitions are possible. In this graph we merely show the most important ones. The transitions shown correspond to all which are (1) more probable than the neutral fixation, and (2) at least 60% as probable as the most frequent one coming out of the same node. Some of the nodes are packed in pairs or trios, due to the fact that separating between them would not contribute with any valuable information about the overall picture. All the transitions coming in or out of the states contained in each of those packs were attributed to the whole. The number of each state follows the numeration of the strategy present in it, which is described in table 2.1. Parameters: $Z = 50$, $\beta = 1$, $a = b = 1$, $p = 0.5$.

Because of the stochastic nature of the evolutionary dynamics, all transitions are possible. In the

graph from fig. 4.2, we merely show the most important transitions coming out of each node. The transitions shown correspond to all which are (1) more probable than the neutral fixation, and (2) at least 60% as probable as the most frequent one coming out of the same node. This simplifying criterion was chosen with the aim of assessing the role that every node has on the dynamics – something central to later understanding the impact of partner choice. Hence our choice to exclude transitions comparing their probabilities to the ones of other transitions coming from the same node, differing from other approaches where nodes with generally rare transitions are ignored from the dynamics [48, 51]. The threshold value of transition exclusion was chosen with the objective of leaving 1 to 4 transitions coming out of each node. Some of the nodes are packed in pairs or trios, due to the fact that separating between them would not contribute with any valuable information about the overall picture. All the transitions coming in or out of the states contained in each of those packs were attributed to the whole.

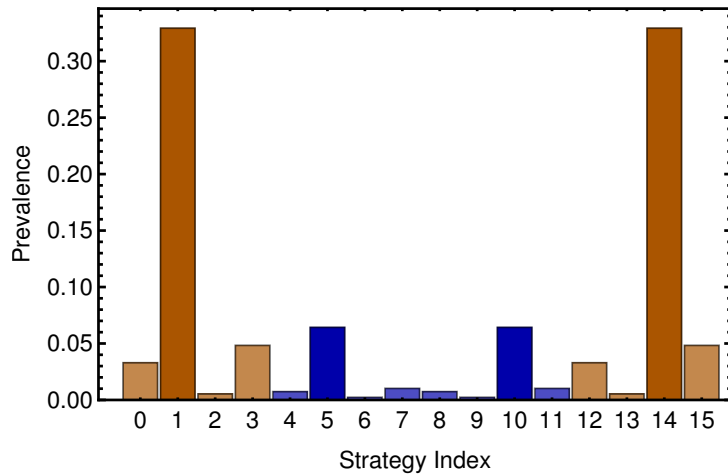


Figure 4.3: Prevalence of each strategy under maximum partial conflict, numbered from 0 to 15. This is a representation of the normalized stationary distribution. Numeration of strategies follows what is described in table 2.1. Strategies from groups I1 and I2 are represented in blue with signaling systems (5 and 10) highlighted in darker blue. Strategies from group UI are represented in brown with deceivers (1 and 14) in darker brown. See graph in fig. 4.2 for a clearer picture of groups. Parameters: $Z = 50$, $\beta = 1$, $a = b = 1$, $p = 0.5$.

Under this analysis, we have chosen to separate the strategies depending on the signals they send, as represented in fig. 4.2: two separate groups of informative strategies – I1 with strategies 4 to 7 and I2 with 8 to 11 – and a third one with all the uninformative – UI. This was done in anticipation of signal-based partner choice, which will be the core of the rest of the analysis. Following table 2.1, we may see that strategies from I1 have the first two digits $\{0, 1\}$, strategies from I2 have $\{1, 0\}$ and strategies from UI have $\{0, 0\}$ or $\{1, 1\}$. This separation according to the signals sent was already made evident by the horizontal lines in that table.

Signaling systems are effectively invaded only by their respective deceptive strategies (fig. 4.2). Besides SS being the second most prevalent strategies (as it was referred in section 4.1 ahead of its time), they are also the only strategies from I1 and I2 that have a significant prevalence (fig. 4.3). We can ascribe this to the fact that all the other strategies in I1 and I2 end up transitioning both to their respective signaling system, and to strategies in UI (fig. 4.2). On a closer look, these transitions happen because,

even though all the strategies from the same informers' group signal the same way, only the signaling systems actually use this information as receivers, and therefore act according to the state of the world, thus receiving the maximum possible payoff.

The dominance of the deceptive strategies (1 and 14) seen in fig. 4.3 seems to come from the whole juncture. First, the strategies most probable to fixate on them (all from UI), lead a near neutral fixation (see section 3.2). Second, the deceptive strategies fixate both inside the UI group and on the group of informers that signals according to their acts (I1 and I2 respectively). On these last groups, not only do they fixate on the SS, as they do it on all the other strategies from that group that, as mentioned already, transition also the SS. They do so by using the information those strategies provide, while not informing them back. This relation seems to be crucial to the understanding of the global dynamics under conflict.

4.3 The effect of partner choice

Now introducing partner choice through the active linking framework, we will explore its effect on the signaling prevalence σ . Our main focus will be signal-based partner choice – section 4.3.1 – but we will nonetheless explore outcome-based partner choice as an alternative – 4.3.2. We should note that all results exhibited further on feature the active linking framework. This way, there is an effect equivalent to a small deviation in intensity of selection β between the results of the previous sections, and the ones obtained for no norm ($\delta = 0$) under this section.

4.3.1 Signal-based partner choice

Let us start by recovering the new curve of σ over a varying conflict level b/a , and see what it looks like when the signal-based norms introduced in section 3.4 are used. This is shown in fig. 4.4.

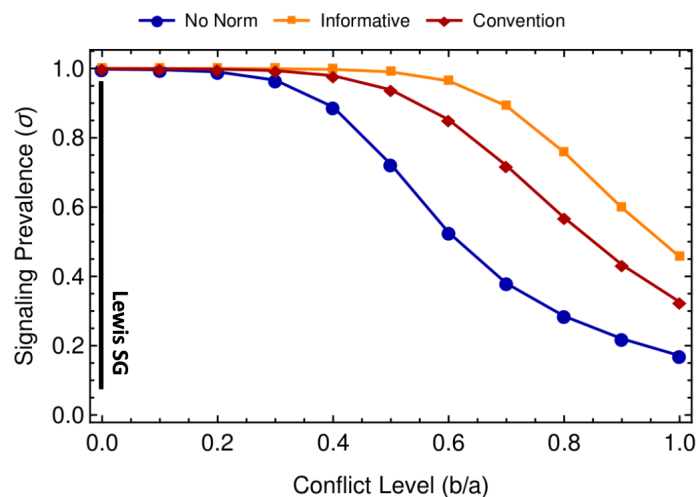


Figure 4.4: Signaling prevalence under signal-based partner choice and changing values of conflict level b/a . The no norm setting was obtained imposing no discrimination ($\delta = 0$). The different values of the conflict level b/a were obtained indirectly through the variation of parameter b . On limit $b/a = 0$, this setting becomes the Lewis signaling game; on limit $b/a = 1$, the level of partial conflict becomes maximum. Other parameters: $\delta = 0.25$, $Z = 50$, $\beta = 1$, $a = 1$, $p = 0.5$.

On that figure, we can see that the Informative and Convention norms systematically hold better results than the no norm scenario. These two norms seem to prolong the plateau coming from the Lewis game where $\sigma > 99\%$ to higher conflict levels than $b/a = 0.5$ and $b/a = 0.3$ respectively. Under the no norm scenario this was only valid for $b/a \leq 0.2$. By examining the fixation probabilities we are able to state that SS are ERS up until conflict levels of $b/a = 0.7$ under both signal-based norms, which in the no norm scenario happened only up to $b/a = 0.4$. There is a decrease in their efficiency in guaranteeing signaling emergence for higher conflict values, but their results are nonetheless positive in relation to the scenario with no partner choice.

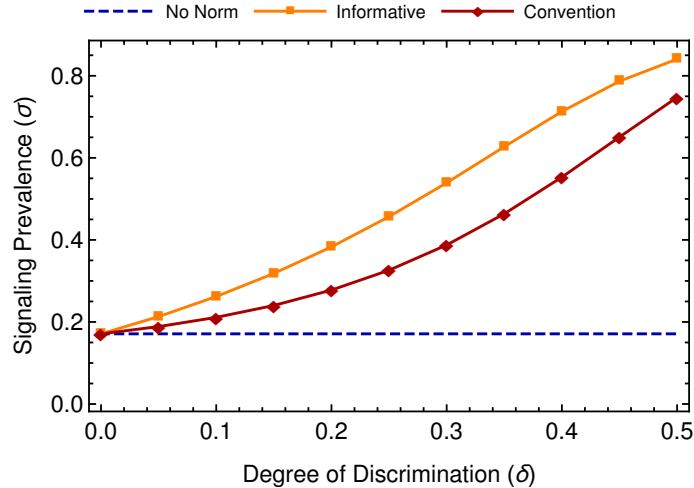


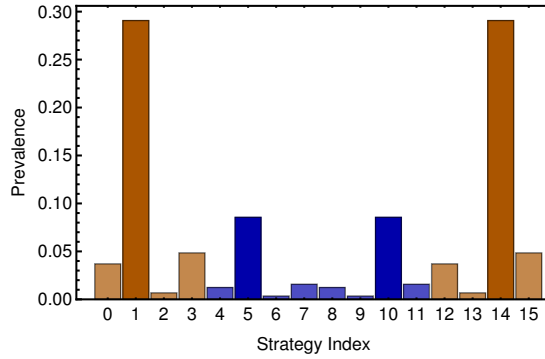
Figure 4.5: Signaling prevalence under signal-based partner choice and changing values of degree of discrimination δ . The no norm setting – dashed line – is shown only as a term of comparison, since it is obtained imposing no discrimination ($\delta = 0$). Other parameters: $Z = 50$, $\beta = 1$, $a = b = 1$, $p = 0.5$.

Now locking our game to the worst-case scenario, where partial conflict is maximum $b/a = 1$, let us see the impact of each of the norms on the signaling prevalence σ , for an increasing degree of discrimination. This is represented in fig. 4.5. As expected, when δ tends to 0, all the norms tend to the same result – $\sigma \approx 17.1\%$ – due to the fact there is no distinction between norms when discrimination is nonexistent. By increasing the degree of discrimination δ , we can see σ increasing up to 84.1% and 74.8% ($\delta = 0.5$) under the Informative and Convention norms respectively. Those two norms show to be consistently leading to the sustenance of higher signaling prevalence. These results are valid for all used values of the degree of discrimination δ , and increasing it only makes the positive effects of the two successful partner choice norms more strongly felt.

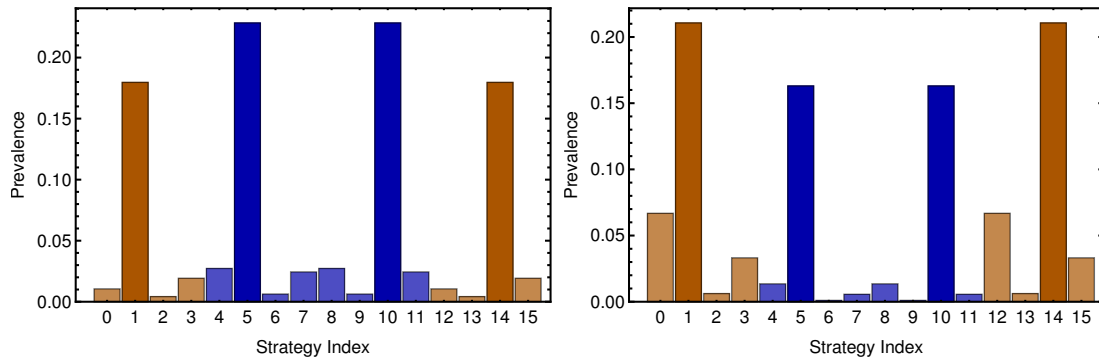
To understand this outcome, we should examine the impact of signal-based partner choice on the dynamics of the system. We will do so by looking at the altered dynamics under a degree of discrimination $\delta = 0.25$ and using as a baseline the one evoked in the previous section 4.2. We focus on the new stationary distributions which are presented in fig. 4.6, and on some of the transitions represented in fig. 4.2, whose probabilities are quantified in table 4.1. All these results are laid in comparison to the no norm scenario – fig. 4.6a and column “No Norm” of table 4.1. We repeat that the no norm scenario under the active linking framework is obtained by choosing a null degree of discrimination $\delta = 0$, therefore exhibiting a change equivalent to a small deviation in intensity of selection β regarding the dynamics of the previous

sections. This is evident comparing figures 4.3 and 4.6a.

It is worth noting that under this formulation of the game there is a symmetry associated to switching the used signals. This makes it that every state has its symmetric state with the same prevalence and with the same fixation probabilities to other symmetric states (see fig. 4.2). Because none of the used norms breaks this symmetry, we can display the fixation probabilities of table 4.1 in pairs of symmetric transitions. **In the following subsections, we will thus refrain from mentioning the effects of the three norms on the symmetric transitions.**



(a) Strategy prevalence under no norm ($\delta = 0$).



(b) Strategy prevalence under Informative norm. (c) Strategy prevalence under Convention norm.

Figure 4.6: Prevalence of each strategy, numbered from 0 to 15, under maximum partial conflict and signal-based partner choice. Three different scenarios are presented: no norm ($\delta = 0$), Informative norm and Convention norm. This is a representation of the normalized stationary distribution. Numeration of strategies follows what is described in table 2.1. Strategies from groups I1 and I2 are represented in blue with signaling systems (5 and 10) highlighted in darker blue. Strategies from group UI are represented in brown with deceivers (1 and 14) in darker brown. See graph in fig. 4.2 for a clearer picture of groups. Parameters: $\delta = 0.25$, $Z = 50$, $\beta = 1$, $a = b = 1$, $p = 0.5$.

Informative norm

The dynamics generated under this norm can be interpreted through the observation of the graph from fig. 4.2 with the values of the fixation probabilities under Informative norm in table 4.1. The resulting stationary distribution is represented in fig. 4.6b.

Comparing fig. 4.6b to fig. 4.6a, we see that the Informative norm allows the two SS to gain a strong prevalence over the two deceptive strategies. Strategies from the I1 and I2, such as 4, 7, 8 and 11 also become more prevalent, when before we could consider them as merely transitory states.

Transition	Fixation Probability ($/\rho_Z$)		
	No Norm	Informative	Convention
5 → 1 10 → 14	7.8	3.8	2.5
6 → 5 9 → 10	14.2	16.5	16.5
6 → 12 9 → 0	13.1	11.5	9.8
6 → 13 9 → 2	18.8	17.6	14.5
6 → 15 9 → 3	13.4	12.1	9.6
4 → 5 8 → 10	7.8	9.2	9.2
4 → 1 8 → 14	7.8	3.8	2.5
7 → 5 11 → 10	7.5	8.9	8.9
7 → 1 11 → 14	7.5	5.9	5.4
1 → 2 14 → 13	1.2	1.1	1.2
1 → 3 14 → 15	1.2	1.1	1.2

Table 4.1: Fixation probabilities of the most relevant transitions between monomorphic states under signal-based partner choice. The number of each state follows the numeration of the strategy present in it, which is described in table 2.1. Strategies are colored according to the group from fig. 4.2 to which they belong: blue for I1 or I2 (informative) and brown for UI (uninformative). All transitions are present in fig. 4.2. The probabilities are displayed for evolution under three different partner choice settings: no norm ($\delta = 0$), Informative norm and Convention norm. The probabilities are normalized to the neutral fixation probability $\rho_Z = 1/Z$ (see section 3.2). Parameters: $\delta = 0.25$, $Z = 50$, $\beta = 1$, $a = b = 1$, $p = 0.5$.

By introducing this norm, links between informers and non-informers are weakened, when compared to the ones between two informers, which are promoted. The second assertion makes sure that transitions within the informative groups I1 and I2 increase in value – 4, 6, 7 → 5 from fig. 4.2 in table 4.1. These transitions occur systematically from other informers to the SS, which secures the SS their favored position in the group.

Let us think about the first assertion now. Looking at fig. 4.2, we see that most of the transitions between informers and non-informers occur **from** the former **to** the later. The Informative norm thus guarantees that the informers that are transitioning with higher probability to the SS, will also transition with less probability to the UI group. This is represented by the decrease in value of the fixation probabilities of 4, 7 → 1 and 6 → 12, 13, 15 – see table 4.1. It will also make sure that the SS themselves will transition less to their respective deceivers, seen in table 4.1 as the decrease of the transition probability of 5 → 1.

Nonetheless, the states of strategies 1 and 14 are still the second most prevalent scenarios. This makes sense, since the Informative norm affects the same way transitions that happen between the same groups, thus leaving the dynamics within the UI, I1 and I2 communities very similar. The fixation on deceivers 1 → 2, 3 is nearly as probable here as it was under no norm.

Summing it up, the Informative norm makes sure that informers tend less to become non-informers (where deceivers are included) and more to become signalers; while leaving signalers more stable and also less prone to become themselves deceivers. By doing this, the norm ensures that signaling will be more than twice as prevalent as it would be without it, under the used choice of parameters.

Convention norm

The dynamics generated under this norm can be interpreted through the observation of the graph from fig. 4.2 with the values of the fixation probabilities under Convention norm in table 4.1. The resulting stationary distribution is represented in fig. 4.6c.

From fig. 4.6c, we can see that some of the features resulting from the use of the Informative norm are replicated under the Convention norm: in comparison to fig. 4.6a, we see the deceptive strategies' prevalence decreasing and the signaling systems' prevalence rising – although neither as intensely as under the Informative norm. Strategies 0 and 12 become surprisingly relevant.

To understand the effect of the Convention norm, we have to think that it translates into an explicit segregation of four different groups: UI1: 0 to 3; I1: 4 to 7; I2: 8 to 11; UI2: 12 to 15. This enables the reinforcement of the dynamics within I1 and I2 and the isolation from the other groups. The alteration of the dynamics under the Convention norm has the same main consequences pointed out under the Informative norm: informers tend less to become non-informers (decrease in probability of $4, 7 \rightarrow 1$ and $6 \rightarrow 12, 13, 15$) and tend more to become signalers (increase in probability of $4, 6, 7 \rightarrow 5$); and signalers tend less to become deceivers (decrease in probability of $5 \rightarrow 1$). All of this can be confirmed in table 4.1.

Even though the Convention norm is similar in some aspects to the Informative one, it differs in a couple of others. First, its effect on the links between informers (from I1 or I2) and non-informers (from UI) is more degrading than the Informative's. This can be seen in table 4.1, where the probabilities of transition between those groups are shown to decrease considerably more under the Convention norm, than under the Informative norm. This being the case, it would be expected that the Convention norm should attain better values of signaling prevalence than the Informative norm, which we see is not happening. This takes us to the second difference.

Under the Convention norm, the UI group is broken into the two mentioned groups (UI1 and UI2), which changes completely the intricate dynamics between the UI states of the population, allowing strategies such as 0 and 12 to rise. These strategies become relevant because they only transitioned to strategies outside their respective group, transitions that almost completely disappear under the Convention norm. The fact that 0 and 12 appear as surprisingly prevalent strategies, explains the lower increase on the prevalence of the signaling systems 5 and 10, when compared to the increase seen under the Informative norm.

4.3.2 An alternative to signal-based partner choice

We will explore the effects of the outcome-based norm introduced in section 3.4 on the emergence of signaling. This form of partner choice relies on distinct information from the encounters when compared to the signal-based forms, thus making us studying it as a potential alternative to the other mechanism. We start by showing in fig. 4.7 the signaling prevalence σ curve for a varying conflict level b/a under this setting, in comparison to the no norm and signal-based scenarios as they are present in fig. 4.4.

On that figure, we can see that outcome-based partner choice systematically holds better results than

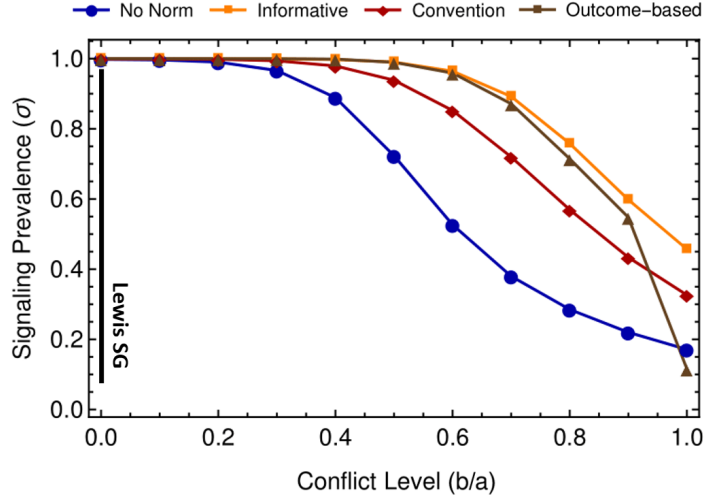


Figure 4.7: Signaling prevalence under outcome-based partner choice and changing values of level of conflict b/a . The no norm setting was obtained imposing no discrimination ($\delta = 0$). Signal-based norms are shown as a term of comparison. The different values of the conflict level b/a were obtained indirectly through the variation of parameter b . On limit $b/a = 0$, this setting becomes the Lewis signaling game; on limit $b/a = 1$, the level of partial conflict becomes maximum. Other parameters: $\delta = 0.25$, $Z = 50$, $\beta = 1$, $a = 1$, $p = 0.5$.

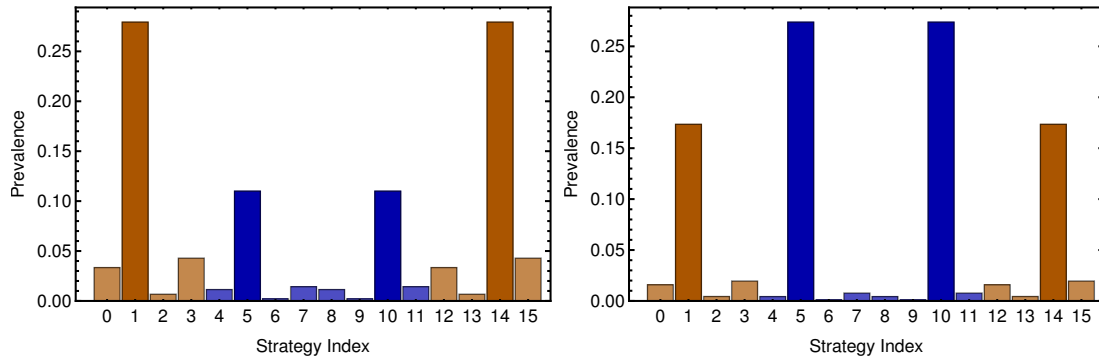
the no norm scenario. Similarly to what happened under signal-based partner choice, the plateau coming from the Lewis game where $\sigma > 99\%$ is prolonged to higher conflict levels up to almost $b/a \approx 0.5$. Directly examining the fixation probabilities we would see that SS are ERS up until conflict levels of $b/a = 0.7$, which in the no norm scenario happened only up to $b/a = 0.4$. Despite assuring higher signaling prevalence values in relation to the scenario with no partner choice, there is a decrease in the outcome-based norm's efficiency in guaranteeing signaling emergence for higher conflict values. For the limit where $b/a = 1$, there is an accentuated decline of σ , and for that specific point, the deceiving strategies become ERS.

To understand these results, we should start by examining the non-trivial linking dynamics developed under this outcome-based norm and the way they couple with the strategic dynamics. Under this setting, an analysis based on the the group division provided in section 4.2 is not as useful, due to the fact this is not a signal-based mechanism and therefore does not affect strategies depending on the signals they use.

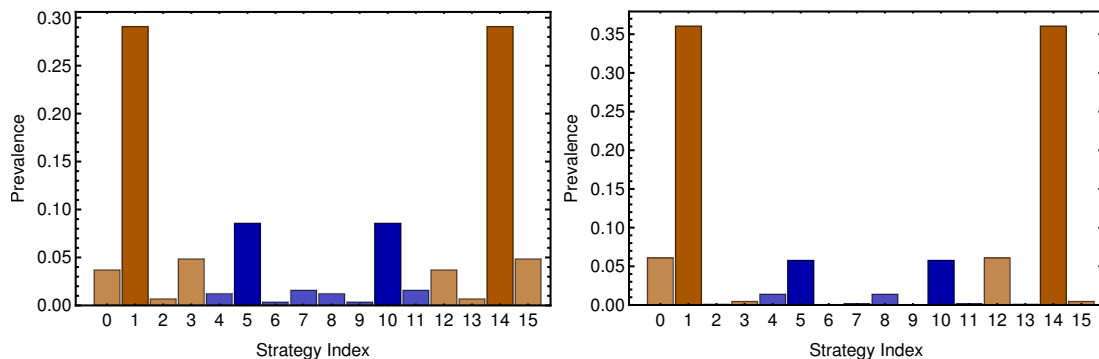
Due to the definition provided, here partner choice depends on the payoffs received by signaling systems, which in turn are defined by both the payoff framework and the incidence of each state of the world – parameters a , b and p . Under equiprobable states ($p = 0.5$), we observe that linking dynamics can assume the three forms characterized by the death rate matrices from appendix's section A.2. Those are correspondent to intervals $0 \leq b/a < 0.5$ (fig. A.4), $0.5 \leq b/a < 1$ (fig. A.5) and $b/a = 1$ (fig. A.6). While the linking dynamics represented by the first two matrices provide a solid boost on signaling prevalence – comparable to the one obtained under the Informative norm – the last one, present in the singular point of maximum conflict $b/a = 1$, holds results worse than the no norm scenario.

This may come as a surprise, but comparing the death rate matrices from that appendix, it should be evident that signaling systems lose their privileged position in the linking dynamics at $b/a = 1$. Under $0 \leq b/a < 1$, signaling systems are the only strategies which have inclinations towards their equals (they

have the only γ_S on the diagonals), coming from the fact that they receive the highest payoffs out of all the monomorphic population. As soon as $b = a$, this is no longer true, since monomorphic states with strategies 0, 1, 4, 8, 12 and 14 (between them the deceivers) hold exactly the same average payoff (see the payoff framework provided in section 3.1). This affects the linking dynamics, allowing populations under maximum conflict level $b/a = 1$ to be considerably more connected without signaling.



(a) Strategy prevalence under no norm ($\delta = 0$) and (b) Strategy prevalence under outcome-based norm and conflict level $b/a = 0.9$.



(c) Strategy prevalence under no norm ($\delta = 0$) and (d) Strategy prevalence under outcome-based norm and conflict level $b/a = 1$.

Figure 4.8: Prevalence of each strategy, numbered from 0 to 15, under high partial conflict and outcome-based partner choice. The no norm scenario ($\delta = 0$) is shown in comparison. Fig. 4.8c is equal to 4.6a and is reproduced here for convenience only. This is a representation of the normalized stationary distribution. Numeration of strategies follows what is described in table 2.1. Strategies from groups I1 and I2 are represented in blue with signaling systems (5 and 10) highlighted in darker blue. Strategies from group UI are represented in brown with deceivers (1 and 14) in darker brown. See graph in fig. 4.2 for a clearer picture of groups. Parameters: $\delta = 0.25$, $Z = 50$, $\beta = 1$, $a = 1$, $b = 0.9, 1$, $p = 0.5$.

The stationary distributions represented in figure 4.8 show exactly how the linking dynamics couple with the strategic ones. We repeat once again that the no norm scenario from figs. 4.8a and 4.8c (this last equal to the one in fig. 4.6a and reproduced here for convenience only) is obtained under the active linking framework using $\delta = 0$ and therefore exhibits a change equivalent to a small deviation in intensity of selection β regarding the results obtained in sections 4.1 and 4.2.

Comparing figs. 4.8a and 4.8b, we see how the preferential linking inclination between signaling systems present under the outcome-based norm for $b/a < 1$ guarantees them to overcome deceivers in the overall dynamics. In fact, looking at the prevalence of each strategy in both scenarios, we see that all but the SS have their prevalence diminished by the introduction of outcome-based partner choice.

When in the maximum conflict level $b/a = 1$, we see the direct result of the SS monomorphic states lacking the privileged spot they had for lower conflict values. All the mentioned connected monomorphic states 0, 1, 4, 8, 12 and 14 (see diagonal of death rate matrix A.6) have their prevalence increased, particularly allowing deceivers to reinforce their prevalence over the dynamics. As shown in figs. 4.8c and 4.8d, the signaling systems still have some relevance on the system's dynamics, but their prevalence decreases in comparison to the no norm scenario, thus becoming the third most common set of strategies, after deceivers (1 and 14) and strategies 0 and 12.

In this analysis of the population dynamics, we conclude that outcome-based partner choice may provide a valuable alternative to the signal-based mechanism already studied. Its effects seem to rely on the fact that under non-maximum partial conflict, signaling systems sustain the higher payoffs out of all the possible monomorphic systems. This means that outcome-based partner choice dynamics assure signaling populations to be more connected, representing a positive reinforcement of signaling emergence. However, maximum partial conflict poses a clear threat to this mechanism, due to the occurrence of other non-SS monomorphic states as efficient in terms of payoff receipt as the SS themselves.

4.4 Final Remarks

In this chapter we have first supplied in section 4.1 an analysis of the effects of partial conflict on the Lewis signaling game. We have concluded that finite populations provide us with an interesting picture, where signaling relevantly occurs at all levels of partial conflict, a result impossible to obtain under deterministic models of infinite population.

Its emergence occurs differently across these levels, which led us to interpret its presence in three different regimes:

1. Complete **dominance** over other states – signaling has a prevalence of over 99% – seen under strong alignment of interests;
2. **Prevalence** with the relevant occurrence of other states – signaling is the most prevalent strategy, even though prevalence values are lower than 99% – seen under weak alignment and weak partial conflict of interests;
3. **Relevance** under the prevalence of deceiving (as will be seen in detail in the next section) – signaling is not the most common strategy, but it has a prevalence of over 5% – seen under strong partial conflict of interests.

This picture was altered by the inclusion of adaptive interaction networks, as done in section 4.3. We showed that the population structure developed under signal-based partner choice, both under the Informative and Convention norms, was rather successful in boosting signaling emergence. These two partner choice norms consistently extended the regime of signaling dominance for larger conflict levels. In some cases they refrained the system under maximum partial conflict from ever getting to the relevance or even the prevalence regime. The robustness of these conclusions is proved in appendix B under a large

range of the parameter space, where the existence of other regimes of effectiveness of those two norms is shown and analyzed.

We have seen that their success should be mainly based on the signal groups described in section 4.2 and encapsulated in fig. 4.2 on which they acted systematically due to them being signal-based. Both norms reinforced links between informers and signaling systems and degraded the links between all of these and the non-informers, where deceivers are included. This led to three changes that we considered crucial:

- Non-SS informers tending less to become non-informers;
- Non-SS informers tending more to become signalers;
- Signalers tending less to become deceivers.

We have further explored the effects of outcome-based partner choice, Because this relied on distinct information from the encounters, we aimed at assessing its value as a potential alternative to signal-based mechanisms. We have seen that it can be as successful as the first under non-maximum partial conflict, due to the fact that signaling systems sustain the higher payoffs out of all the possible monomorphic states, therefore becoming the most connected states. We recognized nonetheless that, because this is no longer true in the extreme scenario where partial conflict is maximum, some complications are expected to arise.

We can be reluctant about all the three stated evolutionary features observed under signal-based partner choice being completely necessary to see the population having the experienced positive outcome under that setting. The reluctance sustaining this question is answered in appendix C, where we explore the outcome of the evolution when only signalers are able to have discriminating linking behavior towards other agents. The results obtained under that appendix were quite expressive. We have concluded that the success of the two signal-based norms requires global action, therefore suggesting the three dynamical features previously stated to be crucial.

We should also be reluctant about the evolutionary origin of partner choice behavior that is sustaining population structure under our model [41]. Sure the results provide an optimistic outcome regarding signaling emergence, but can we justify the development of the linking behavior defined by partner choice, solely guided by the agents' individual benefit? This is answered in the next chapter 5, where we provide an analysis of the evolution of those norms in a population where adopters and non-adopters of each norm may be present in the same population. This should provide a good measure of the general evolutionary likelihood of each of the partner choice norms, while testing the robustness of the emergence of signaling under the co-evolution of linking behavior.

Chapter 5

Co-evolution of Signaling and Partner Choice Norms

In this chapter, we will study the evolutionary origins of partner choice and its co-emergence with signaling. To do so, we will overlap dynamics of different linking behavior on top of the already handled strategic and network dynamics. Behavioral dynamics arise from allowing different behavioral types to coexist and evolve in a dynamical population. This means that agents should be indeed allowed to interact with others in distinct ways. This happens not only on each encounter, as it is defined by their strategy, but also as they are forming links with other agents. Under this work, we intend to study one particular setting defined by the coexistence of agents which do not discriminate on their linking inclinations and therefore choose to link with every agent, with agents which link according to one globally-defined partner choice norm. The purpose of this setting is to explore the likelihood of one partner choice norm being evolutionarily adopted by a population.

In section 5.1, we extend our framework, outlining a suitable implementation. The subsequent discussion of results is initiated in section 5.2, where we focus on the evolutionary likelihood of each of the two proposed signal-based partner choice norms. In section 5.3, we move on to explore the impact that the coexistence of partner choice behavior has on the emergence of signaling. The chapter ends in section 5.4, with a review of all the conclusions drawn on it.

5.1 Model

Agents using the same strategy may be distinguished by linking according to one partner choice norm, or by using maximum partner choice and therefore being inclined to link with everyone. We will respectively call them adopters (A) and non-adopters (NA) of the partner choice norm at hand. Under this evolutionary setting there are 32 types of agents. Each type is defined by the combination of its strategy and its linking inclinations. There are 16 different strategies – the same we had up until now – and 2 linking possibilities – the agent is either using the partner choice norm at hand or she is not.

We should start by elaborating on the linking dynamics which will occur under this setting. We have

defined partner choice norms in section 3.4 as being based on each agent’s inclination to link with other agents. By assessing the inclination that every pair of types of agents had towards each other, we were able to define the death rate matrix, crucial under linking dynamics.

Here we follow the same approach and extend it to the 32 existing types of agents. We should be able to compute the 32×32 death rate matrix. The numeration of agents, from 0 to 31, follows the strategy numeration done in table 2.1, leaving 0 – 15 to represent all non-adopters of the norm, and 16 – 31 its adopters. The NA are inclined towards all types of agents, while the A have their inclinations defined by the norm chosen to be studied. Evaluating the inclination of every possible pair of types of agents, and following the already defined procedure to translate this into the death rate of their links, we get the complex 32×32 death rate matrix. Combining this with fixing every propensity value at 1, we have the linking dynamics completely defined.

Let us move on to the stochastic process, which is essentially kept the same, except for the inclusion of the new types of agents. Selection now happens between the 32 types, and when mutating, agents may do it with equal probability $1/(2n_s - 1) = 1/31$ into all the other combinations of $\{strategy, norm\}$. The transition matrix M^{ev} between monomorphic states under the SML has to be extended to the 32×32 possible transitions, and the resulting stationary distribution φ^{ev} (eq. 5.1) will have likewise 32 entries. **All the vector and matrix elements are numbered from 0 to 31**, following the same order as the one defined under the death rate matrix.

$$\varphi^{ev} = \varphi^{ev} M^{ev} \quad (5.1)$$

To get the norm prevalence under this evolutionary setting, we sum over the prevalence of each monomorphic state which uses the norm (eq. 5.2).

$$P(norm) = \sum_{k=16}^{31} \varphi_k^{ev} \quad (5.2)$$

In order to obtain signaling prevalence among non-adopters, we compute eq. 5.3. Signaling prevalence among adopters is obtained by eq. 5.4. And general signaling prevalence is obtained through eq. 5.5.

$$P(signal|NA) = (\varphi_5^{ev} + \varphi_{10}^{ev}) \cdot \left[\sum_{k=0}^{15} \varphi_k^{ev} \right]^{-1} \quad (5.3)$$

$$P(signal|A) = (\varphi_{21}^{ev} + \varphi_{26}^{ev}) \cdot \left[\sum_{k=16}^{31} \varphi_k^{ev} \right]^{-1} \quad (5.4)$$

$$P(signal) = \varphi_5^{ev} + \varphi_{10}^{ev} + \varphi_{21}^{ev} + \varphi_{26}^{ev} \quad (5.5)$$

The way we have defined norms in section 3.4, we see that not being inclined towards other agents directly implies that some of the encounters that that agent would have will not occur. This determines that by linking according to a norm, an agent’s payoff against every other agent is either reduced or kept constant, in comparison to what that agent would receive if inclined towards everyone. Consequently,

we say that adopting partner choice represents an evolutionary cost to a population. It should be noted that a different formulation, where the payoffs received are not constrained to positive values [54] could be a way around this cost. If receivers failing to match their actions with the state of the world were attributed a negative payoff, avoiding uninformative agents could translate into very high benefits.

This cost may however be justified, since undergoing it must likewise represent a cost to the other agents with whom they miss encounters. In a sense, this is similar to what we see in [75, Ch. 5.3, p. 112], where a finite population analysis is done, adding an extra strategy who interacts based on a different game with all other agents. This strategy pays a cost (has a reduced payoff when compared to the equivalent strategy under the unaltered game) in order to assure a punishment (reduction in payoff again) to a selection of other strategies. This cost may be worth adopting by a population, if the punishment it inflicts on other agents decreases their chances of fixating.

Looking at [52, 56], we can find a methodology close to the one used here. There we similarly see agents that use the same strategy having different linking behaviors. The definition of the active linking parameters is parallel to the one we do in section 3.4. The implicit norm that is used by the “fast” agents (FC and FD) is an inclination towards cooperators and avoidance of defectors. Its results show that the only defectors relevant in the evolutionary dynamics are the less discriminating ones, while for the cooperators the opposite happens. Then, from our standpoint, we would be able to say that cooperators undergo the cost of adopting that form of partner choice, while defectors do not.

We intend to evaluate if agents under the signaling game undergo the evolutionary cost of adopting partner choice, as it has been previously studied. Additionally, we intend to explore the impact of the coexistence of partner choice behavior and indiscriminate linking agents on the emergence of signaling. The evolutionary dynamics under this extension of the model will be analyzed under the same standard choice of free parameters. Nevertheless, the robustness of the conclusions drawn in the next sections was assessed under the wider range of the parameter space exhibited in table 5.1.

Parameter	Symbol	Range
Population Size	Z	{25, 50 , 100}
Intensity of Selection	β	{0.5, 1 , 5}
Game Parameter a	a	{1}
Game Parameter b	b	{0.6, 0.9, 1 }
Conflict Level	b/a	{ b/a }
Probability of state of the world S_0	p	{0.4, 0.5 , 0.6}
Degree of Discrimination	δ	{0, 0.25 , 0.5}
Linking Propensity	α	{1}
Slow death rate	γ_S	{0.5 - δ }
Fast death rate	γ_F	{0.5 + δ }
Medium death rate	γ_M	{0.5}

Table 5.1: Parameters present in the extended model and parameter space analyzed. The values highlighted in the range represent the standard ones being displayed in the next sections.

5.2 Evolutionary likelihood of partner choice norms

Studying the evolution of the two signal-based partner choice norms individually against non-adopters, and computing norm prevalence according to eq. 5.2, we have obtained fig. 5.1.

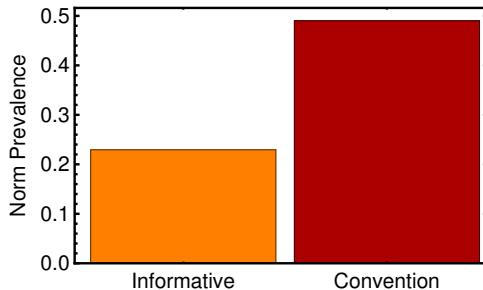


Figure 5.1: Norm prevalence under the extended model with possible coexistence of agents that link based on a partner choice norm (adopters – A), and those equally inclined towards all other agents (non-adopters – NA). Each bar results from the independent evolutionary dynamics of that partner choice norm. Parameters: $\delta = 0.25$, $Z = 50$, $\beta = 1$, $a = b = 1$, $p = 0.5$.

From this figure we should start by pointing out that both norms have a significant presence when evolved, which is a fairly positive outcome. Looking in detail, we can see that under the evolution of the Convention norm, adopters are approximately as prevalent as non-adopters. This does not seem to happen under the evolution of the Informative norm, where adopters have a lower probability of being found.

A good justification for this to happen should be the fact that the Informative norm is universal, as defined in section 3.4, in a way that leaves half of the strategies having no inclination towards agents using those same strategies. This happens to all the strategies from UI, and leaves the population in their respective monomorphic states more disconnected. It seems highly probable that these strategies do not benefit from adopting this form of partner choice, and paying the inherent cost to do it, therefore leaving this norm generally less evolutionarily plausible.

The Convention norm on the other side, still holds good results – more than two times the incidence of the Informative norm under evolution. This leads us to conclude that the Convention norm has a remarkable evolutionary plausibility, distinguishing it from the Informative one, which adds to the already promising results from the last chapter.

It is interesting to point out that outcome-based partner choice, as it is introduced in section 3.4 holds signaling prevalence values similar to those of the Informative norm under $b/a < 1$. This similarity should be related to the fact that both norms cause non-informative states to be less connected, a mechanism which seems to reduce the chances of those norms being adopted by them. On the other side, for $b/a = 1$ the outcome-based norm evolved with a surprisingly high prevalence, systematically higher than 50%, and therefore higher than the signal-based ones. Despite that limiting form of partner choice not affecting the global dynamics in a way that favors signaling systems (as seen in the previous chapter), it is still a very effective one to be evolutionarily adopted by populations.

These conclusions hold for the parameter space exhibited in table 5.1. We have observed the Convention norm having a consistent prevalence between 45% and 55%, while the Informative norm (and

similarly the outcome-based one for $b/a < 1$) showed to be the less prevalent, especially under higher intensity of selection β or population size Z values.

5.3 Co-emergence of signaling and partner choice

We will make an effort to understand the way signaling emerges under these new evolutionary circumstances. Let us start by looking at the prevalence of signaling systems under evolving norms. In fig. 5.2, we can see the results obtained for the two signal-based norms compared with the ones obtained in the previous chapter, where norms were fixed. An additional dashed line is present, representing the prevalence of SS under the no norm scenario where $\delta = 0$.

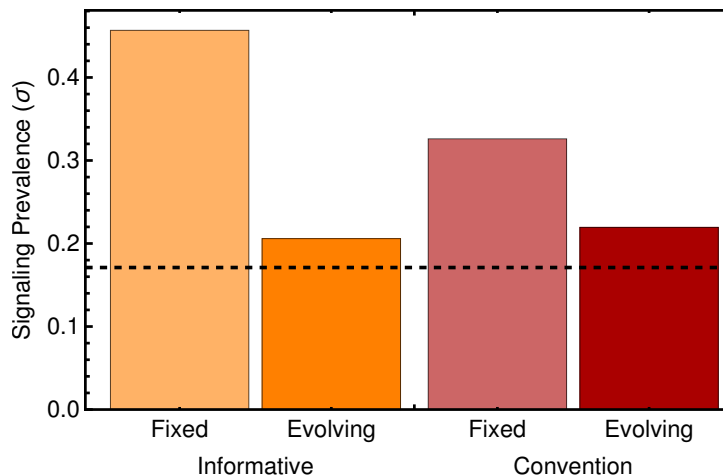


Figure 5.2: Comparison between the signaling prevalence results obtained under the initial model – where each norm is **fixed** – and the extended model with possible coexistence of different partner choice behavior – where each norm can **evolve**. The value obtained under the no norm setting ($\delta = 0$ in the initial model) is shown in a dashed line as a term of comparison. Parameters: $\delta = 0.25$, $Z = 50$, $\beta = 1$, $a = b = 1$, $p = 0.5$.

We can observe that the Convention and Informative norms hold considerably better signaling results when fixed than when evolving. This might be due to the fact these norms sustain signaling more effectively than the no norm scenario, as we can see by comparing the dashed line with the fixed columns of these two norms. The signaling prevalence under the evolving norms is showed to fall between the respective fixed norm and the no norm values.

It is interesting to point out that even though the Informative norm held better results than the Convention one when fixed, the same does not happen when those norms are evolving. The Convention norm shows to be as effective sustaining signaling as the Informative one, or even more. Nonetheless, both evolving norms were shown to systematically increase signaling prevalence under the whole free parameter space from table 5.1, in comparison to the respective no norm scenarios. The increase provided under the evolving Informative norm was additionally observed to be extremely sensitive to higher values of intensity of selection β or population size Z . This could be linked to two results already pointed out: the decrease of the evolutionary likelihood of the Informative norm, seen in section 5.2; and the lower signaling prevalence under the fixed Informative norm in those limits, analyzed in the appendix B.

In order to understand the previous results regarding norm and signaling prevalence, it should be interesting to observe the presence of signaling among adopters and non-adopter of the partner choice norms. This can be interpreted as an indicator of whether the used norm favors signaling systems in particular or not. The discrepancy between the signaler’s presence among non-adopters and adopters is the larger the more signaler-centered those norms are. In fig. 5.3 we quantify the prevalence of signaling separately among adopters and non-adopters of the two signal-based partner choice norms.

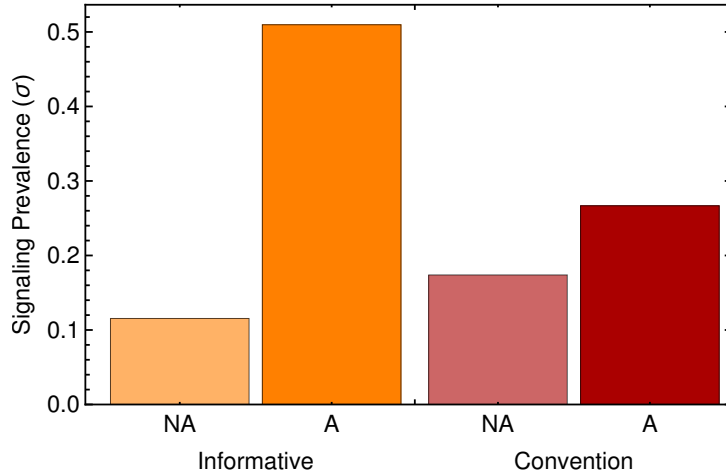


Figure 5.3: Comparison of signaling prevalence among adopters (A) and non-adopters (NA) of signal-based partner choice. Results were obtained from the extended model with possible coexistence of different partner choice behavior. The independent evolutionary dynamics under each of the norms contemplates both agents discriminating others following it (adopters), and agents equally inclined towards all other agents (non-adopters). Parameters: $\delta = 0.25$, $Z = 50$, $\beta = 1$, $a = b = 1$, $p = 0.5$.

Comparing the two values for the Informative norm, we see that more than half of the norm adopters are signalers, a value that drops significantly among non-adopters. The fact that the Informative norm clearly favors signalers disproportionately, has as a consequence that signalers are a large fraction of the few strategies that undergo the cost of adopting partner choice. This makes it a generally less evolutionarily plausible norm, as seen in section 5.2, and lowers its impact on the overall signaling prevalence 20.6%.

The Convention norm has an effect similar to the Informative’s, but in a clearly smaller scale. This norm still favors signalers, as it should, but not in such an extreme form, making signaling prevalence have a lower discrepancy among adopters and non-adopters. This is also connected to the higher general evolutionary plausibility of the norm, which allows it to attain higher prevalence values than the Informative norm, as it was seen in section 5.2. This improvement is also felt on the slightly higher general signaling prevalence obtained when the norm is evolving: 21.9% in fig. 5.2.

This discrepancy between values of signaling among adopters and among non-adopters under the Informative and Convention norms was observed across the whole parameter space exhibited in table 5.1.

We have additionally tested signaling values under the previously introduced outcome-based norm. We have observed again that for $b/a < 1$, the norm behaves similarly to the Informative norm. Signaling under its mechanism was frail to the coexistence of norm adopters and types inclined to link to all, showing a decrease to values generally lower than under the Convention norm and a high discrepancy

between values among adopters and non-adopters of the norm. Under the limit $b/a = 1$, this norm had a fairly different outcome with signaling values close to the no norm scenario, both among adopters and non-adopters of the norm. This should be enough evidence of the fact that in that limit, this norm does not favour signalers more than it favours the other relevant strategies.

5.4 Final Remarks

In this chapter we studied the evolutionary likelihood of partner choice, focusing on the signal-based norms introduced in section 3.4. Additionally, we probed the role that co-evolving linking behavior had on the emergence of signaling. The conclusions drawn from the obtained results are summed up in the following points.

- We have stated in section 5.1 that in order for populations to adopt partner choice, and therefore sustain structure under the current model, they would have to undergo an evolutionary cost. The results from section 5.2 could not be clearer in showing that this cost is paid, and that population structure indeed emerges. This conclusion directly addresses the concerns expressed in [41], about the origin and maintenance of population structure from an evolutionary perspective.
- The Convention norm held prevalence values under our model twice as high as the Informative norm, showing to have the highest evolutionary likelihood. This could be associated with the fact that the Informative norm has a special focus on the particular success of informative strategies. This should cause populations in non-informative states to be more disconnected, which should prevent them from adopting the norm, and eventually lead to the discrepancy in signaling values among adopters and among non-adopters. Additional support for this assertion should come from the fact that outcome-based partner choice held consistently similar results to the Informative norm, showing that this characteristic present in both settings could be determining the outcome of the extended model. Nonetheless, we conclude that the general emergence of the Convention norm was particularly positive, adding to the promising results it had in the previous chapter. This comes to show that besides partner choice being apt to evolve, it is also attainable that the particular structure evolving from it sustains signaling.
- Signaling prevalence is always increased under the evolving Convention and Informative norm, in comparison to the outcome obtained under a population with indiscriminate partner choice. Even though this boost was lighter than the one obtained when linking behavior defined by those same norms was fixed, it was consistently observed all over the parameter space from table 5.1. Based on this, we assert that the enhancement of signaling in the present model is moderately robust to the co-evolution of the linking behavior creating population structure. As previously stated, an alternative formulation of the payoff matrix with negative values could limit the evolutionary cost of partner choice, possibly showing more robust results [54].

Chapter 6

Conclusions

6.1 Achievements

We now recall the objectives designed to be achieved under the current work and elaborate on what was concluded under each of them:

- **Does signaling emerge under partial conflict?**

The proposed unifying payoff framework allowed us to include partial conflict in the Lewis signaling game and study its effect on signaling emergence. We have shown the occurrence of three distinct regimes: 1) complete dominance over other states – seen under strong alignment of interests; 2) prevalence with the relevant occurrence of other states – seen under weak alignment and weak partial conflict of interests; 3) relevance under the prevalence of deceiving – seen under strong partial conflict of interests. This may serve to conclude that, contrary to what infinite population models tell us, signaling can be at least evolutionarily relevant at all conflict levels.

- **What are the main mechanisms affecting the emergence of signaling?**

Using stochastic finite populations showed us a rather complex picture of the dynamics, which would not be possible under deterministic dynamics. In order to provide a clarification of the results obtained in the limiting scenario of maximum partial conflict, we have divided the 16 strategies over 3 groups – two groups of informative strategies I1 and I2, differentiated by how they inform, and the uninformative ones UI. The main points attained were: non-SS informative strategies tend to become signalers from their group, as well as uninformative ones; signalers transition mainly to deceivers; and uninformative strategies transition between them. This should explain the overall prevalence of strategies from UI – especially deceivers – and the still relevant prevalence of SS.

- **Can time-evolving population structure generated by partner choice enhance the emergence of signaling?**

We have introduced partner choice as a way of handily develop population structure under adaptive interaction networks. We have shown that signal-based partner choice – through the Informative

and Convention norms – was consistently successful in boosting signaling prevalence under all levels of partial conflict. Both norms extended the dominance of signaling to higher levels of conflict, and in some particular cases, allowed the system to never leave that regime. Increasing the agents' level of discrimination under signal-based partner choice consistently enhanced signaling prevalence. This result is undoubtedly conclusive, since it correlates the depth of the changes felt in the population structure with the emergence of signaling.

We have provided a solid interpretation of this success essentially based on three effects that the changes in structure provoked on the signal groups: non-SS informers tending less to become non-informers; non-SS informers tending more to become signalers; signalers tending less to become deceivers. We assessed the relevance of all features being present by reducing partner choice behavior to signalers, concluding that the norms required global action in order to be successful.

We have shown that outcome-based partner choice can be an alternative effective in reinforcing signaling emergence. The success of this mechanism relies on signaling systems holding the highest payoffs out of all the possible monomorphic systems (outside the maximum partial conflict limit), guaranteeing they become the most connected and consequently stable of them.

- **Can partner choice emerge from natural selection?**

By enabling the co-evolution of partner choice with the present strategic and network dynamics, we were able to assess the evolutionary origins of this behavior. The results could not be clearer in showing that the evolutionary cost of partner choice was paid by populations, which translated into the development of population structure. The Convention norm held fairly good results. This is particularly promising, since it adds that not only is population structure likely to emerge, as it is attainable that the particular structure evolving will enhance signaling. Even though the signaling enhancement enabled by the Informative and Convention norms was mitigated by their co-evolution, it still emerged consistently. The co-emergence of structure and signaling, comes to show the evolutionary robustness of the positive effects structure has on signaling.

The designed model additionally introduced two completely novel features. One of them was the signaling game and its unifying payoff framework, where the conflict level was used to gauge continuously from the Lewis SG to the canonical SG with partial conflict. The other feature was partner choice, which was central to the use of active linking and allowed us to model adaptive interaction networks under the complete non-truncated 16-strategy dynamics in finite populations. The extension of this model aiming at the study of evolutionary linking behavior was therefore also novel, as well as the idea of cost of adopting partner choice drawn from that extension.

We ultimately hope this work is a valuable contribution to the existing literature on signaling. Hitherto, a large part of it has been dedicated to costly signaling as a mechanism to overcome deceiving behavior under partial conflict. We have deliberately tried to diversify this landscape by bringing attention to other features of real populations, such as finiteness and structure, and showing them to have remarkably promising effects on the emergence of signaling.

6.2 Future Work

The present work was designed and performed with a general approach, resorting to the minimum required assumptions. We thus encourage the extension of the used model as well as the application of its conclusions to unforeseen systems. Next are proposed new research prospects arising from this work and possibly being carried out in the future.

It should be interesting to explore alternative forms of linking behavior, possibly focused on the actions performed by agents, the resemblances between them, or the formation of cliques. Each norm's logical complexity [96, 97] could be assessed formally, as this factor could play a role in the norms' success in promoting signaling [98]. If the computational expenses are overcome, partner choice could be additionally explored subjecting all its possible combinations to natural selection at the same level [100], or under a multi-level selection model [101]. These models could provide valuable insights about both the evolutionary likelihood of this behavior and the co-emergence of signaling. Separating strategic mutations from linking behavior ones by providing each with its independent probability under the co-evolution setting may alternatively provide distinct emerging outcomes under the evolutionary dynamics [102].

Two major analytic approximations were used under this model: the small mutation limit [12, 103] and fast network dynamics under active linking [54, 55]. Both approximations could have their validity assessed using agent-based numerical simulations to evaluate changes in the outcome. As an example, this is done under the prisoner's dilemma in [52] for $\mu = Z^{-2}$, and in [55] for linking dynamics under a wide range of τ_a/τ_s . Even though results are shown to hold in both cases, it is not trivial that the same should happen under signaling game dynamics. The SML could be further extended including additional configurations of interest apart from the monomorphic states, to achieve more accurate results [53].

Together with [41], the present work shows that population structure is a theoretically viable and solid alternative to costly signaling theory. We recognize the gap between these results and the experimental data. Therefore, we urge for the need to connect data on signaling from real complex systems from biological and social sciences, with the theoretical results drawn here. This could be done looking for structure in those systems and understanding how it can enhance signaling emergence based on what is shown here, or alternatively using the theoretical framework developed under this work to study the impact of specific network dynamics.

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Appendix A

Partner choice norm definition

In this appendix we provide a representation of different used active linking's death rate matrices. In section A.1, we define the ones obtained under each of the signal-based partner choice norms introduced in section 3.4. In section A.2, we provide the three representations possible under the outcome-based norm from section 3.4 for $p = 0.5$. We then move on to do the same thing in section A.3, for the signalers-exclusive version of norms, which are used in the appendix C. Let us start by recovering table 2.1.

Index	Representation	Name
0	{0, 0, 0, 0}	Deceiver 1
1	{0, 0, 0, 1}	
2	{0, 0, 1, 0}	
3	{0, 0, 1, 1}	
4	{0, 1, 0, 0}	Signaling System 1
5	{0, 1, 0, 1}	
6	{0, 1, 1, 0}	
7	{0, 1, 1, 1}	
8	{1, 0, 0, 0}	Signaling System 2
9	{1, 0, 0, 1}	
10	{1, 0, 1, 0}	
11	{1, 0, 1, 1}	
12	{1, 1, 0, 0}	Deceiver 2
13	{1, 1, 0, 1}	
14	{1, 1, 1, 0}	
15	{1, 1, 1, 1}	

Table A.1: Numbered strategies and their representation. Deceivers are shown in brown and signaling systems in blue. Their representation was obtained the following way: {signal sent when state of the world is S_0 ; signal sent when state of the world is S_1 ; act performed when signal received is M_0 ; act performed when signal received is M_1 }. Additionally, strategies are separated by horizontal lines according to the signals sent (first two digits of their representation). This should be useful when using the group picture introduced in section 4.2.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	γ_S	γ_S	γ_M	γ_M	γ_S	γ_S	γ_M	γ_M	γ_S	γ_S	γ_M	γ_M	γ_S	γ_S	γ_M	γ_M
1	γ_S	γ_S	γ_M	γ_M	γ_M	γ_M	γ_F	γ_F	γ_F	γ_F	γ_F	γ_F	γ_M	γ_M	γ_F	γ_F
2	γ_M	γ_M	γ_F	γ_F	γ_F	γ_F	γ_F	γ_F	γ_M	γ_M	γ_F	γ_F	γ_S	γ_S	γ_M	γ_M
3	γ_M	γ_M	γ_F	γ_F	γ_M	γ_M	γ_F	γ_F	γ_M	γ_M	γ_F	γ_F	γ_M	γ_M	γ_F	γ_F
4	γ_S	γ_M	γ_F	γ_M	γ_S	γ_M	γ_F	γ_M	γ_S	γ_M	γ_F	γ_M	γ_S	γ_M	γ_F	γ_M
5	γ_S	γ_M	γ_F	γ_M	γ_M	γ_S	γ_M	γ_F	γ_F	γ_M	γ_F	γ_F	γ_M	γ_F	γ_F	γ_F
6	γ_M	γ_F	γ_F	γ_F	γ_F	γ_M	γ_F	γ_F	γ_M	γ_S	γ_M	γ_F	γ_S	γ_M	γ_F	γ_M
7	γ_M	γ_F	γ_F	γ_F	γ_M	γ_F	γ_F	γ_F	γ_M	γ_F	γ_F	γ_F	γ_M	γ_F	γ_F	γ_F
8	γ_S	γ_F	γ_M	γ_M	γ_S	γ_F	γ_M	γ_M	γ_S	γ_F	γ_M	γ_M	γ_S	γ_F	γ_M	γ_M
9	γ_S	γ_F	γ_M	γ_M	γ_M	γ_M	γ_S	γ_F	γ_F	γ_F	γ_M	γ_F	γ_M	γ_F	γ_F	γ_F
10	γ_M	γ_F	γ_F	γ_F	γ_F	γ_F	γ_M	γ_F	γ_M	γ_M	γ_S	γ_F	γ_S	γ_F	γ_M	γ_M
11	γ_M	γ_F	γ_F	γ_F	γ_M	γ_F	γ_F	γ_F	γ_M	γ_F	γ_F	γ_F	γ_M	γ_F	γ_F	γ_F
12	γ_S	γ_M	γ_S	γ_M	γ_S	γ_M	γ_S	γ_M	γ_S	γ_M	γ_S	γ_M	γ_S	γ_M	γ_S	γ_M
13	γ_S	γ_M	γ_S	γ_M	γ_M	γ_F	γ_M	γ_F	γ_F	γ_F	γ_F	γ_F	γ_M	γ_F	γ_M	γ_F
14	γ_M	γ_F	γ_M	γ_F	γ_F	γ_F	γ_F	γ_F	γ_M	γ_F	γ_M	γ_F	γ_S	γ_M	γ_S	γ_M
15	γ_M	γ_F	γ_M	γ_F	γ_M	γ_F	γ_M	γ_F	γ_M	γ_F	γ_M	γ_F	γ_M	γ_F	γ_M	γ_F

Table A.6: Death rate matrix under outcome-based partner choice norm. Parameters: $a = b = 1$, $p = 0.5$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S
1	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S
2	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S
3	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S
4	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S
5	γ_M	γ_M	γ_M	γ_M	γ_S	γ_S	γ_S	γ_S	γ_M	γ_M	γ_F	γ_M	γ_M	γ_M	γ_M	γ_M
6	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S
7	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S
8	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S
9	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S
10	γ_M	γ_M	γ_M	γ_M	γ_M	γ_F	γ_M	γ_M	γ_S	γ_S	γ_S	γ_S	γ_M	γ_M	γ_M	γ_M
11	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S	γ_S
12	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S
13	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S
14	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S
15	γ_S	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_M	γ_S	γ_S	γ_S	γ_S	γ_S

Table A.9: Death rate matrix under signaler-exclusive Convention partner choice norm.

Appendix B

Robustness of success of signal-based partner choice

In this appendix, we will test the validity of the results previously stated in section 4.3.1. We will mainly focus on probing the success of the Informative and Convention norms in increasing the signaling prevalence σ relating to the no norm scenario. When the results obtained under the two partner choice norms differ, we will seek to understand the source of their differences.

In order to do so, we resorted to testing variations of the free parameters β , Z and p under different choices of δ , whose ranges are listed in table 3.1, and then analyzed the effects these variations had on the dynamics' results.

The fixation probability of deceivers on signaling systems showed to be a relevant value in the analysis. In table B.1 we display this probability for different choices of parameters, which will be used ahead. The concept of evolutionarily robust strategy (ERS) will also come up frequently, thus being advised a revision of its definition in section 3.2.

Regime	Fixation Probability ($/\rho_Z$) of $5 \rightarrow 1 / 10 \rightarrow 14$ under		
	No Norm	Informative	Convention
$\delta = 0.25, Z = 50, \beta = 1, p = 0.5$	7.8	3.8	2.5
$\delta = 0.25, Z = 50, \beta = 100, p = 0.5$	50.0	50.0	0.2
$\delta = 0.5, Z = 50, \beta = 1, p = 0.5$	7.8	0.4	0.3
$\delta = 0.5, Z = 50, \beta = 100, p = 0.5$	50.0	< 0.1	< 0.1
$\delta = 0.25, Z = 50, \beta = 1, p = 0.5$	7.8	3.8	2.5
$\delta = 0.25, Z = 250, \beta = 1, p = 0.5$	38.5	16.7	3.6
$\delta = 0.5, Z = 50, \beta = 1, p = 0.5$	7.8	0.4	0.3
$\delta = 0.5, Z = 250, \beta = 1, p = 0.5$	38.5	< 0.1	< 0.1
$\delta = 0.25, Z = 50, \beta = 1, p = 0$	14.4	12.1	9.0
$\delta = 0.25, Z = 50, \beta = 1, p = 0.5$	7.8	3.8	2.5
$\delta = 0.25, Z = 50, \beta = 1, p = 1$	1.0	< 0.1	0.1

Table B.1: Fixation probability of deceivers on signaling systems ($5 \rightarrow 1 / 10 \rightarrow 14$) for different values of parameters. These probabilities are displayed for evolution under three different partner choice settings: no norm ($\delta = 0$), Informative norm and Convention norm. The probabilities are normalized to the neutral fixation probability $\rho_Z = 1/Z$ (see section 3.2). Other parameters: $a = b = 1$.

B.1 Effect of intensity of selection β

We have firstly tested the robustness of the results for variations of the intensity of selection β . We show the signaling prevalence σ in fig. B.1, across the range $0.05 \leq \beta \leq 200$, for both $\delta = 0.25$ and $\delta = 0.5$, fixing all the other parameters $Z = 50$, $a = b = 1$, $p = 0.5$.

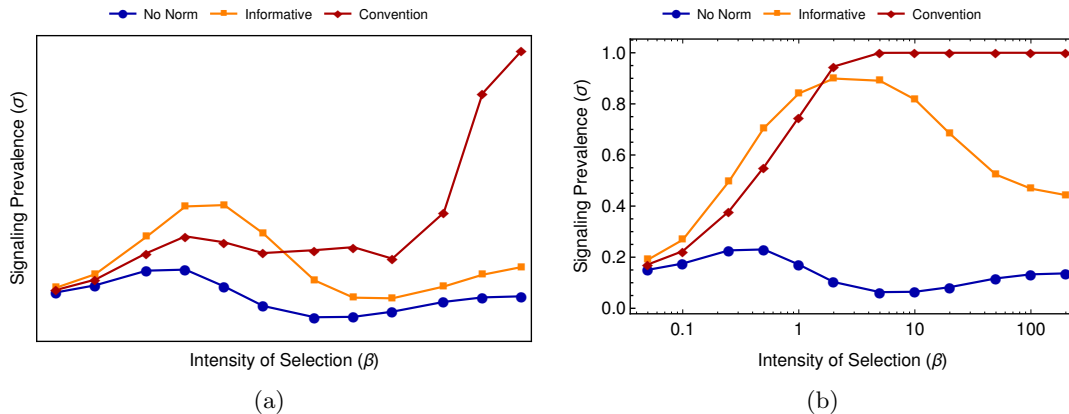


Figure B.1: Signaling prevalence under signal-based partner choice and changing values of intensity of selection β . The no norm setting was obtained imposing no discrimination ($\delta = 0$). The two images differ on the degree of discrimination used under the norms: $\delta = 0.25$ in fig. B.1a and $\delta = 0.5$ in fig. B.1b. Other parameters: $Z = 50$, $a = b = 1$, $p = 0.5$.

We see similar profiles in both figures B.1. The no norm curves are the same, due to the fact that a change in the degree of discrimination δ does not affect the results of the non-discriminative scenario. Signaling prevalence under the Informative norm shows a peak around average values of the intensity of selection in both cases. The Convention norm holds satisfying values of signaling prevalence for average values of β . For high β we get a clear boost on its value, almost up to its maximum of $\sigma \approx 100\%$.

We separate this analysis into two regimes: the average intensity of selection β , and the high intensity of selection β , which we see here for the first time. Let us first focus on the average β regime.

In both figures B.1, this regime, defined by the range where the conclusions previously stated hold, is limited by the peak seen under the Informative norm. When $\delta = 0.25$ (fig. B.1a), this peak occurs roughly in the interval $0.25 < \beta < 2$. For $\delta = 0.5$ (fig. B.1b) we can see it being enlarged, both achieving a maximum signaling prevalence $\sigma \approx 90\%$, and representing an overall boost in that value in comparison to the no norm and the $\delta = 0.25$ scenarios, for the whole range studied here $0.05 < \beta \leq 200$.

Turning into the high β regime, we define it in the range where the signaling prevalence under the Convention norm boosts almost into its maximum value. We see that under $\delta = 0.25$ (fig. B.1a) this starts around $\beta \approx 100$, and under $\delta = 0.5$ (fig. B.1b), it starts around $\beta \approx 2$.

By increasing the impact of the norms – through the degree of discrimination δ upon which they act – we widen the Informative norm peak and anticipate the values of β at which the Convention norm blows up. We partially merge the two regimes, bringing out the promising results present in each of the two.

We can now ask the following question: where is the difference between the two norms under the high β regime rooted? In that limit, we observe that there is something peculiar happening with the system under the Convention norm: 4 strategies emerge as evolutionarily robust strategies (ERS) – strategies

3, 5, 10 and 15 – meaning that no strategy fixates on those with higher probability than $1/Z$ – value corresponding to the neutral fixation.

As we have considered when drawing fig. 4.2, the only strategies relevantly fixating on the signaling systems when $\beta = 1$ are the deceptive strategies. When we move into high values of β , those transitions can either almost disappear – as they do under the Convention norm – or escalate to their maximum value – as they do under no norm and the Informative norm (see table B.1).

In order to understand why such a great difference occurs, we have to look further into the transitions between the SS and their respective deceptive strategies. In figure B.2 we can see the gradient of selection of those transitions for $\beta = 1$ and $\beta = 100$. The Convention norm creates a small negative gradient area, whose limits do not seem to change by increasing β , but whose depth gets amplified. Note that the $\beta = 1$ panels (top) have a $\times 10$ **scale effect** in comparison to the $\beta = 100$ ones (bottom). This means that the negative gradient pit under the Convention norm and $\beta = 100$ is much larger than it seems. This is the reason why deceivers will not be able to fixate on signaling systems under that norm. The Informative norm does not affect the profile of the gradient in this way, leaving it positive all over k . This is enough to allow the SS to become ERS under the Convention norm and not under the Informative norm.

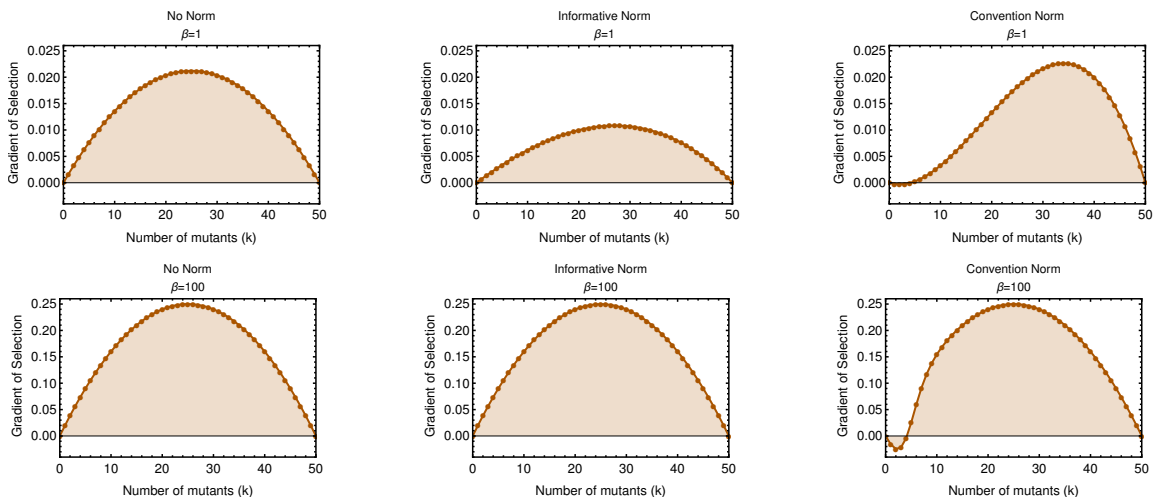


Figure B.2: Gradient of selection of deceivers in a population of signaling systems. The top panels have a $\times 10$ **scale effect** in comparison to the bottom, due to the discrepancy in values. The computations were performed with a population comprised of $Z - k$ signalers and k mutants using the respective deceiving strategy. Explicitly, the gradient of selection is $T^+(k) - T^-(k)$ (see eq. 3.4). In this case it represents the difference between the probability of one signaler being converted into a deceiver and the probability of the inverse happening. The three columns of plots represent the gradients of selection under no norm $\delta = 0$ (first), the Informative norm (second) and the Convention norm (third). The two rows of plots represent different intensities of selection $\beta = 1$ (top) and $\beta = 100$ (bottom). Parameters: $\delta = 0.25$, $Z = 50$, $a = b = 1$, $p = 0.5$.

Under $\delta = 0.5$ the picture is a bit different. Looking at the values in table B.1 correspondent to average $\beta = 1$ and high $\beta = 100$ under that degree of discrimination, we see that the transition probabilities under both the Informative and Convention norm are smaller than the one of the neutral drift. However, in fig. B.1b, we easily see that signaling never gets to completely dominate the dynamics under the Informative norm. That is because another transition becomes relevant and eventually overthrows both signaling systems, which is $5 \rightarrow 9$ and $10 \rightarrow 6$. This does not happen under the Convention, where SS become

again ERS, due their discrimination towards those strategies from other groups.

Summarizing, we have seen that the conclusions about the success of the two signal-based norms hold for average values of the intensity of selection. There is a clear high β regime, where the dynamics change considerably and the Convention norm holds systematically better results. This regime is attributed to SS becoming ERS under the Convention norm, due to their segregation being 1) more effective towards deceivers and 2) enlarged to all strategies outside their group. The two regimes get less delimited for high degrees of discrimination, and in that case both norms hold exceptionally good results. Increasing the degree of discrimination is shown again to systematically boost the prevalence of signaling under both norms.

B.2 Effect of population size Z

We move on to the robustness of the success observed under signal-based partner choice for variations of the population size Z . We show the signaling prevalence σ in fig. B.3, across the range $10 \leq Z \leq 250$, for both $\delta = 0.25$ and $\delta = 0.5$, fixing all the other parameters $\beta = 1$, $a = b = 1$, $p = 0.5$.

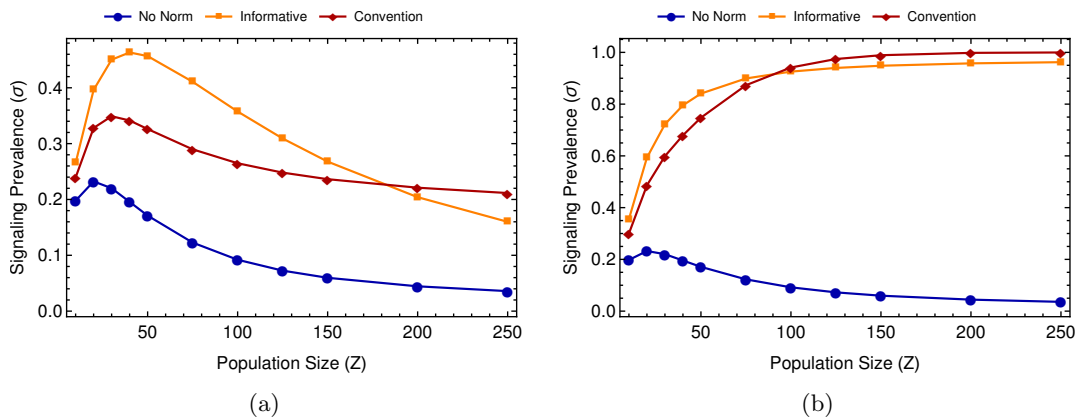


Figure B.3: Signaling prevalence under signal-based partner choice and changing values of population size. The no norm setting was obtained imposing no discrimination ($\delta = 0$). The two images differ on the degree of discrimination used under the norms: $\delta = 0.25$ in fig. B.3a and $\delta = 0.5$ in fig. B.3b. Other parameters: $\beta = 1$, $a = b = 1$, $p = 0.5$.

Here we can see an effect similar to the one obtained changing the intensity of selection β . There are two regimes, as may be observed in the two figures B.3a and B.3b. For an average degree of discrimination $\delta = 0.25$ (fig. B.3a), we see a peak of signaling prevalence σ for average values $10 \leq Z \leq 50$, and a subsequent decrease for higher values of Z . This peak occurs under all three tested scenarios: no norm, Informative norm and Convention norm. It comes to show that signaling emerges at higher degrees under signal-based partner choice for average sized populations, when the degree of discrimination is average.

By moving to the maximum degree of discrimination $\delta = 0.5$ (fig. B.3b), we can see that the behavior of the Informative and Convention norms becomes rather similar and both convey a solid boost on signaling prevalence that attains $\sigma \approx 1$. This dominance was already observed under coinciding interests and is here successfully reproduced under conflict.

Looking at table B.1, we can see that, again, the fixation probability of the deceivers on their signaling

systems may convey information about the observed outcomes. This probability does not vanish under high Z , when $\delta = 0.25$, meaning that deceivers will keep fixating on SS, preventing these last from having any dominance over the dynamics. This is not true under $\delta = 0.5$, where the fixation probabilities are always lower than that of the neutral drift for average $Z = 50$, but get almost null for high $Z = 250$. Signaling systems still do not become ERS under the Informative norm, due to minor transitions to strategies from the other informative group ($5 \rightarrow 9$ and $10 \rightarrow 6$). However, the lack of any other transition reveals them as completely dominant strategies under both the Informative and Convention norms, for $\delta = 0.5$ and high Z .

Summarizing, the effect of population size Z shows to be comparable to that of intensity of selection β . Different signaling regimes are obtained. The one under which the previous dynamics hold comes up for average values of population size and an average degree of discrimination, where the Convention and Informative norms moderately boost signaling prevalence. Under higher discrimination levels, both successful norms present exceptional results, that consistently improve for increasing population size Z . Intensifying the degree of discrimination is shown again to systematically boost the prevalence of signaling under both norms.

B.3 Effect of probability p

Here we study the effect that the probability p of occurrence of state S_0 has on the signaling prevalence σ under signal-based partner choice. We show this in fig. B.4, across the possible values $0 \leq p \leq 1$, fixing all the other parameters $\delta = 0.25$, $Z = 50$, $\beta = 1$, $a = b = 1$.

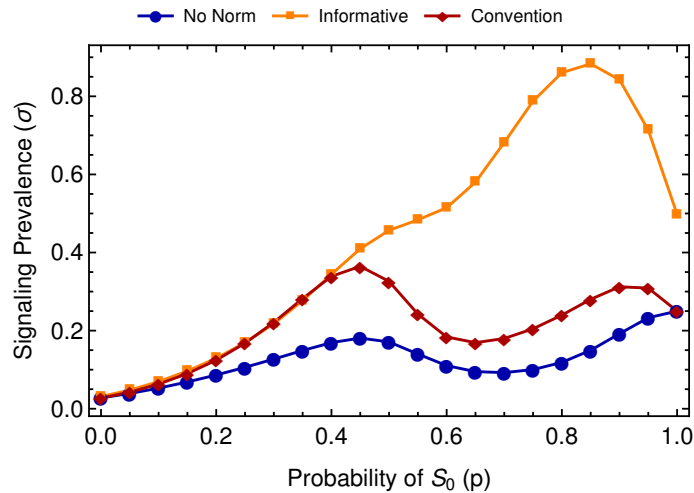


Figure B.4: Signaling prevalence under signal-based partner choice and changing values of probability p of state of the world S_0 . The no norm setting was obtained imposing no discrimination ($\delta = 0$). Other parameters: $\delta = 0.25$, $Z = 50$, $\beta = 1$, $a = b = 1$.

We have stated in section 4.1 that the system did not seem sensitive to deviations in p under a well-mixed population. Here we see that when we have partner choice norms breaking those grounds, the system becomes particularly more sensitive.

Under this setting, we can see a region close to $p = 0.5$ – value used up until now – where the

previously stated results hold. That region is the one with the most significance to our analysis, due to both states S_0 and S_1 being present with substantial probabilities.

Looking at that region – the average p regime – we can see a pattern common to the Convention norm and the no norm scenarios, defined by the shape of a peak. This peak is more prominent under the Convention norm, which should reflect its success in guaranteeing the emergence of signaling. The Informative norm holds signaling prevalence values almost coincident with the ones under the Convention norm for $p \leq 0.45$. At this value, σ under the Informative receives a boost, and the results of the two leading norms diverge.

To understand why that divergence occurs, we need to step outside the equiprobable regime. By moving $p \rightarrow 1$, the game becomes progressively reduced to state S_0 , thus becoming more and more cooperative. This has a positive impact on the emergence of SS under the Informative norm, scenario which allows them to be ERS from $p = 0.75$ on (see table B.1), and their prevalence to attain maximum values of $\sigma \approx 88\%$. For higher values of p , strategies 8 and 10 also become ERS, thus lowering the signaling prevalence. The Convention norm never attains such values of σ because there are several other strategies that become ERS, such as the deceivers 1 and 14, before SS are able to do it.

By moving in the opposite direction $p \rightarrow 0$, the game becomes progressively reduced to state S_1 , thus becoming increasingly competitive. Under this limit, there is no chance for SS to become ERS, due to their transition to deceiving strategies, seen in table B.1. In this case, and under every scenario, only uninformative strategies become ERS, therefore decreasing signaling prevalence until it becomes $\sigma \approx 0$.

The existence of several ERS when deviating from the equiprobable regime can be assigned to the simplification of the game. Different strategies receive payoffs increasingly similar, eventually limited to 0, 0.5 or 1 under $a = b = 1$, in the two extreme cases. In these scenarios, for a monomorphic state to be relevantly prevalent throughout the dynamics, it has to become an ERS, which leads to stationary distributions consisting only on ERS with equal probabilities. Hence the σ experienced in the limits: $\sigma \approx 0$ for $p = 0$ under all scenarios; $\sigma \approx 1/4$ for $p = 1$ under the Convention norm and no norm; $\sigma \approx 1/2$ for $p = 1$ under the Informative norm.

Summarizing, both signal-based norms keep promoting signaling emergence all over the parameter p space. There is clearly an effectiveness peak of the Convention norm around $p = 0.45$, that contains the area of larger significance to our analysis. The Informative norm proved to be equally effective for values of p below the peak, and exceptionally successful for values above.

Appendix C

Signaler-exclusive partner choice

In section 4.3.1, we have tried to provide an explanation of the success of the the two signal-based norms – Informative and Convention – which relied on their global effects. Three features were pointed out as crucial to their mechanism, which we restate here:

- Non-SS informers tending less to become non-informers;
- Non-SS informers tending more to become signalers;
- Signalers tending less to become deceivers.

The study of population structure in signaling games with partial conflict was done in [41] without resorting to these means. It resorts to an infinite population model under which the answers are fairly straightforward: either signaling is able to become stable, or it does not. By considering solely a truncated game between signalers and their deceivers, and therefore the linking dynamics between these two strategies, it is concluded that SS become stable under a relevant fraction of the parameter space.

In this section, we intend to assess the effect on the emergence of signaling when we allow the norms to affect solely those links between SS and all other strategies or, more exclusively, when only the links between SS and their deceivers are affected. This is a way of testing if the experienced success of the Informative and Convention norms requires them to be globally used, and indirectly, the robustness of the three features proposed in section 4.3.1 as crucial.

To do so, we apply here the same norms used in section 4.3.1, but taking only into consideration the partner choice performed by signalers. To keep the intuition around our formulation of the norms, we have defined the other strategies as being inclined towards all. This converts the previously defined global norms into signaler-exclusive norms, which are represented in the appendix's section A.3. We additionally introduce norm SS-D, defined in that same section, which is characterized by having all links but the SS towards their respective deceivers positively inclined. The signaling prevalence obtained under these norms is displayed in table C.1.

The SS-D norm has as its only feature the third one itemized before: signalers tending less to become deceivers. And again, this can be linked to what we see being studied in [41]. It is shown here that it holds mediocre results under our evolutionary model.

	Signaling Prevalence (σ)	
No Norm ($\delta = 0$)	17.1%	
Norms	Signaler-Exclusive	Global
SS-D	22.1%	–
Informative Norm	19.5%	45.7%
Convention Norm	21.5%	32.6%

Table C.1: Signaling prevalence under signaler-exclusive norms (section A.3) and comparison with the results obtained under the global signal-based norms (section A.1). Parameters: $\delta = 0.25$, $Z = 50$, $\beta = 1$, $a = b = 1$, $p = 0.5$.

Under the signaler-exclusive Informative and Convention norms, we do not have present the first feature previously stated: non-SS informers tending less to become non-informers. Both are shown to hold a rather mediocre signaling prevalence increase, similarly to the SS-D norm, when compared to the increase obtained by the Informative and Convention norms acting globally.

Based on these rather expressive results, we assert the importance of global action under the Informative and Convention norms. This is required for the mechanism on which their high success relies to be efficient. This suggests that all the three features previously stated should be crucial, including the one not present in none of these signaler-exclusive norms: non-SS informers tending less to become non-informers.